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**Code No. : 6839** **Sub. Code : PMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Second Semester

Mathematics — Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The equation of the rectifying plane at a point $u$ of the circular Helix $r(u) = (a\cos u, a\sin u, bu)$ is

   (a) $X\cos u + Y\sin u - a = 0$

   (b) $X\cos u + Y\sin u + a = 0$

   (c) $X\cos u - Y\sin u + a = 0$

   (d) $X\cos u + Y\sin u - b = 0$

2. The curvature of the circle $x^2 + y^2 = 25$ is

(a) 0 (b) 5

(c) 25 (d) 115

3. A necessary and sufficient condition for a curve to be a Helix is

(a) curvature is a constant

(b) torsion is a constant

(c) torsion is zero

(d) ratio of curvature to torsion is constant

4. The position vector of the center of the oscillating sphere is

(a) $c = r + \rho n + \sigma b$ (b) $c = r + \rho n + \tau b$

(c) $c = r + \rho n + \rho' \sigma b$ (d) $c = r + \rho n + \rho' \tau b$

5. The direction coefficients of the parametric directions are respectively

(a) $\left(1/\sqrt{E}, 0\right), \left(0, 1/\sqrt{G}\right)$

(b) $\left(1/E, 0\right), \left(0, 1/G\right)$

(c) $\left(-1/\sqrt{E}, 0\right), \left(0, 1/\sqrt{G}\right)$

(d) $\left(1/\sqrt{E}, 0\right), \left(0, -1/\sqrt{G}\right)$

6. If $r = (u, v, u^2 - v^2)$ is the position vector of any point on the paraboloid, then the value of $H^2$ is

(a) $1 + 4u^2 + 4v^2$ (b) $1 - 4u^2 + 4v^2$

(c) $1 + 4u^2 - 4v^2$ (d) $1 - 4u^2 - 4v^2$

7. Every space curve is a geodesic on its

(a) rectifying developable

(b) osculating developable

(c) polar developable

(d) ellipsoid

8. If $\Gamma_{ijk}$, $i, j, k = 1, 2$ are Christoffel symbols of the first kind, then $\Gamma_{111}$ is

(a) $E_1 / 2$ (b) $E_2 / 2$

(c) $G_1 / 2$ (d) $G_2 / 2$

9. If $L$, $M$, $N$ vanish at all points on a surface, then the surface is a

(a) right helicoid (b) sphere

(c) cone (d) plane

10. The Gaussian curvature of the surface $r = (a(u+v), b(u-v), uv)$ is

(a) $4a^2b^2 / H^4$ (b) $-4a^2b^2 / H^4$

(c) $4a^2b^4 / H^4$ (d) $-4a^2b^4 / H^4$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $u$ is the parameter of the curve $r$, then the equation of the oscillating plane at any point $P$ with position vector $\overline{r} = \overline{r}(u)$ is $[R - r, \dot{r}, \ddot{r}] = 0$.

Or

(b) Find the curvature and torsion of $r = (a\cos\theta, a\sin\theta, a\theta\cos\alpha)$.

12. (a) If $R$ is the radius of spherical curvature, show that $R = \dfrac{|t \times t''|}{k^2\tau}$.

Or

(b) Find the involutes and evolutes of the circular helix $r = (a\cos\theta, a\sin\theta, b\theta)$.

13. (a) For a right helicoid given by $(u\cos v,\, u\sin v,\, av)$, determine $(r_1, r_2, N)$ at a point on the surface and the direction of the parametric curves. Find the direction making angle $\frac{\pi}{2}$ at a point on the surface with the parametric curve $v = \text{constant}$.

Or

(b) Prove that position vector of a point on the anchor ring is

$r = ((b + a\cos u), \cos v, (b + a\cos u)\sin v, a\sin u))$

where $(b, 0, 0)$ is the center of the circle and $z$-axis is the axis of rotation.

14. (a) For a variable direction at $P$, prove that $\left|\frac{d\phi}{dS}\right|$ is maximum in a direction orthogonal to the curve $\phi(u, v) = \text{constant}$ through $P$ and the angle between $(-\phi_2, \phi_1)$ and the orthogonal direction in which $\phi$ is increasing is $\frac{\pi}{2}$.

Or

(b) Prove that the curves of the family $\frac{v^3}{u^2} = \text{constant}$ are geodesics on a surface with the metric $v^2du^2 - 2uvdudv + 2u^2dv^2$, $u > 0$, $v > 0$.

15. (a) Prove that a curve on a surface is a geodesic if and only if the geodesic curvature vector is zero.

Or

(b) Show that the points of the paraboloid $r = (u\cos v, u\sin v, u^2)$ are elliptic but the points of the helicoids $r = (u\cos v, u\sin v, av)$ are hyperbolic.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let $\gamma$ be a curve of class $m \geq 2$ with arc length $s$ as parameter. If the point $p$ on $\gamma$ has parameter zero prove that the equation of the oscillating plane is $[R - r(0), r'(0), r''(0)] = 0$ where $r'' \neq 0$. If $r'' = 0$, assuming $\gamma$ is analytic and prove that the equation of the plane at an inflexional point is $[R - r(0), r'(0), r^{(k)}(0)] = 0$.

Or

(b) Prove by an example that at a point of inflexion, a curve of class $\alpha$ need not possess on oscillating plane.

17. (a) Prove that the curvature $k_1$ and torsion $\tau_1$ of an involute $\overline{C}$ of $c$ are $k_1^2 = \frac{\tau^2 + k^2}{k^2(c-s)^2}$, $\tau_1 = \frac{k\tau' - k'\tau}{k(c-s)(k^2+\tau^2)}$.

Or

(b) Find the center of spherical curvature of the curve given by $r = (a\cos u, a\sin u, a\cos 2u)$.

18. (a) Prove that the first fundamental form of a surface is a positive definite quadratic for in $du$, $dv$.

Or

(b) Obtain the surface equation of sphere and find the singularities, parametric curves, tangent plane at a point and the surface normal.

19. (a) State and prove Liouville's Formula.

Or

(b) A helicoids is generated by a screw motion of a straight line which met the axis at an angle $\alpha$. Find the orthogonal trajectories of the generators. Find also the metric of the surface referred to the generators and their orthogonal trajectories as parametric curves.

20. (a) If $k$ is the normal curvature in a direction making an angle $\psi$ with the principal direction $v = \text{constant}$, the prove that $k = k_a \cos^2 \psi + k_b \sin^2 \psi$ where $k_a$ and $k_b$ are principal curvatures at the point $P$ on the surface.

Or

(b) State and prove Rodrigue's formula.