(7 pages) **Reg. No. :** .....

**Code No. : 33001 E** **Sub. Code : AMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2020.

First Semester

Mathematics — Core

CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The curvature of the curve $\frac{x}{a}+\frac{y}{b}=1$ is

(a) 1 (b) 2

(c) 0 (d) none of the above

2. The centre of curvature of $x^2 + y^2 = r^2$ is

(a) $(0, 0)$ (b) $(1, 1)$

(c) $(2, 2)$ (d) none of the above

3. $\int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} r d\theta dr =$ ————————

(a) $\pi$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) none of the above

4. The value of $\left|\frac{1}{4}\right| \left|\frac{3}{4}\right| =$ ————————

(a) $2\pi$ (b) $\frac{\pi}{2}$

(c) $\sqrt{2}\pi$ (d) none of the above

5. The value of $\int_0^{\infty} e^{-x} x^3 dx$

(a) 6 (b) $\frac{1}{6}$

(c) 12 (d) none of the above

6. $\int_0^a \int_0^x \int_0^y xy\,z\,dz\,dz\,ydx =$ ————————

(a) $\frac{1}{48}$ (b) $\frac{a^6}{48}$

(c) $\frac{a}{48}$ (d) none of the above

7. The sum of the squares of the roots of $x^3 - 6x^2 + 11x - 6 = 0$ is

(a) 194 (b) 192

(c) 136 (d) none of the above

8. For the equation $x^4 + px^2 + qx + r = 0 (r \neq 0)$, the value of $s_{-1}$ is

(a) $\frac{r}{q}$ (b) $\frac{r}{p}$

(c) $\frac{-q}{r}$ (d) none of the above

9. The equation whose roots are roots of $x^5 + 6x^4 + 6x^3 - 7x^2 + 2x - 1 = 0$ with the signs changed is

(a) $x^5 - 6x^4 + 6x^3 + 7x^2 + 2x + 1 = 0$

(b) $x^5 - 6x^4 + 6x^3 + 7x^2 + 2x - 1 = 0$

(c) $x^5 + 6x^4 - 6x^3 - 7x^2 - 2x + 1 = 0$

(d) none of the above

10. The equation obtained by multiplying the roots of the equation $x^4 + 2x^3 + 4x^2 + 6x + 8 = 0$ by $\frac{1}{2}$ is

(a) $4x^4 + 4x^3 + 4x^2 + 3x + 2 = 0$

(b) $4x^4 + 4x^3 + 4x^2 + 3x - 2 = 0$

(c) $x^4 + x^3 + x^2 + 4x + 4 = 0$

(d) None of the above

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Find the radius of curvature of $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ at $(1, 2)$.

Or

(b) Find the curvature of $r = (1 - \cos\theta)$.

12. (a) By Changing the order of integration, evaluate $\int_0^3 \int_1^{\sqrt{4-y}} dxdy$.

Or

(b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz\, dy\, dx$.

13. (a) Evaluate $\int_0^1 x^3 (1-x^2)^{5/2} dx$.

Or

(b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{(\sin^2 x)^{1/3}}{\sqrt{\cos x}} dx$.

14. (a) Find the value of $\sum \alpha^2 \beta, \sum \alpha^2 \beta\gamma$ of the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$.

Or

(b) For any equation with real coefficients, prove that the complex roots occur in conjugate pairs.

15. (a) Show that the equation $x^9 - x^5 + x^4 + x^2 + 1 = 0$ has only one real roots which is negative.

Or

(b) Transform the equation $3x^4 - \frac{5}{2}x^3 + \frac{7}{6}x^2 - x + \frac{7}{18} = 0$ into another with integral coefficients and for the coefficient of the first term unity.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Find the $p-r$ equation of $r^m = a^m \cos m\theta$.

Or

(b) Show that in the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$, $x+y=3(x+y)$ where $(x, y)$-centre of curvature.

17. (a) Find the volume of the integration $\iiint_D \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ where $D$ is the region bounded by the planes $x=0, y=0, z=0$ and $x+y+z=1$.

Or

(b) By changing into polar coordinates evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{xdydx}{x^2+y^2}$.

18. (a) Prove that $2^{2n-1}\lceil(n)\lceil(n+1/2)=\lceil(2n)\lceil\pi$.

Or

(b) Using gamma function, show that $\iiint_D \frac{dx\,dy\,dz}{(1-x^2-y^2-z^2)^{\frac{1}{2}}} = \frac{\pi^2}{8}$ where the integration is taken over the region of space in the positive octant bounded by the sphere $x^2 + y^2 + z^2 = 1$.

19. (a) Show that the sum of the $m^{\text{th}}$ power of the roots of $x^n - 2x^{n-1} - 2x^{n-2} - \ldots - 2x - 2 = 0$ is $3^m - 1$ where $m \le n$.

Or

(b) Solve $3x^4 - 40x^3 + 130x^2 - 120x + 27 = 0$ given that the roots are in geometric progression.

20. (a) Solve the equation

$x^6 + 2x^5 + 2x^4 - 2x^2 - 2x - 1 = 0$.

Or

(b) If $\alpha, \beta, \gamma$ are the roots of $x^3 - 3x + 1 = 0$, find the equations whose roots are (i) $\frac{\alpha}{4}, \frac{\beta}{4}, \frac{\gamma}{4}$ (ii) $-\alpha^2, -\beta^2, -\gamma^2$ (iii) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ (iv) $\alpha + 3, \beta + 3, \gamma + 3$ (v) $\alpha - 2, \beta - 2, \gamma - 2$.

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