(7 Pages)

Reg. No. : ....

Code No. : 30346 E   Sub. Code : SMMA 63

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Main

GRAPH THEORY

(For those who joined in July 2017 onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The number of edges in a cubic graph with 8 vertices is

   (a) 8   (b) 24

   (c) 12   (d) 7

2. $K_m + K_n =$

   (a) K_{mn} (b) $K_{m,n}$

   (c) $K_{n,m}$ (d) M_{m+n}

3. The graph corresponding to the partition (2, 1, 1) is ————

   (a) K_3 (b) $K_{1,2}$

   (c) $K_{2,1,1}$ (d) $\overline{K}_3$

4. If ————, then G has a path of length K.

   (a) $\delta \geq K$ (b) $\delta < K$

   (c) $\Delta < K$ (d) $\Delta \geq K$

5. Every Hamiltonian graph is ———— connected.

   (a) 2   (b) 3

   (c) p (d) $p-1$

6. The number of edges in a forest with p vertices and K components is ————

   (a) $p-1$ (b) $p-K$

   (c) $p-K-1$ (d) $p-K+1$

7. Which of the following is not planar?

   (a) K_4 (b) $K_{2,2}$

   (c) $K_{2,5}$ (d) $K_{3,4}$

8. $\chi(C_{101}) = $ —————

(a) 2

(b) 101

(c) 100

(d) 3

9. If G is a (p,q)-graph and $f(G,\lambda) = \lambda^4 - 3\lambda^3 + 3\lambda^2 - \lambda$, then $q = $ —————

(a) 3

(b) 4

(c) 1

(d) 0

10. If the underlying graph of a digraph is connected, then the digraph is called ————— connected.

(a) strongly

(b) weakly

(c) unilaterally

(d) directly

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove : $\Gamma(G) = \Gamma(\overline{G})$.

Or

(b) Prove : $\alpha + \beta = p$.

12. (a) Verify whether the following partitions are graphical. If yes, draw the corresponding graph.

(i) (5, 3, 2, 1, 1, 1, 1, 1, 1)

(ii) (6, 6, 5, 4, 3, 3, 1).

Or

(b) State and prove a necessary and sufficient condition for a line in a connected graph to be a bridge.

13. (a) If G is a graph with $p \geq 3$ vertices and $\delta \geq \dfrac{p}{2}$, then show that G is hamiltonian.

Or

(b) Prove that every tree has a center consisting of either one point or two adjacent points.

14. (a) If any (p,q)- connected plane graph $(p \geq 3)$ with r faces, show that $q \geq \dfrac{3r}{2}$ and $q \leq 3p - 6$.
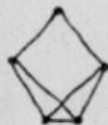
Or

(b) Show that every uniquely n-colourable graph is $(n-1)$-connected.

15. (a) Find the chromatic polynomial of the following graph.

Or

(a) In a directed graph, define the directed walk, directed path and directed cycle. Also give examples.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let G_1 be a (p_1, q_1)-graph and G_2 be a (p_2, q_2)-graph. Then prove :

   (i) $G_1 + G_2$ is a $(p_1 + p_2, \ q_1 + q_2 + p_1 p_2)$-graph

   (ii) $G_1 \times G_2$ is a $(p_1 p_2, \ q_1 p_2 + q_2 p_1)$-graph.

Or

(b) Explain the matrices related with a graph.

17. (a) Prove that a graph with at least tow points is bipartite if and only if all its cycles are of even length.

Or

(b) Prove

   (i) In a graph G, every $u - v$ walk contains a $u - v$ path.

   (ii) If G is a K-connected graph, then $q \geq \dfrac{pK}{2}$.

18. (a) State and prove the Chavatal's theorem on Hamiltonian graph.

Or

(b) Show that the following statements are equivalent for a connected graph G.

   (i) G is eulerian

   (ii) Every point of G has even degree

   (iii) The set of edges of G can be partitioned into cycles.

19. (a) State and prove Euler's formula for a connected plane graph. Also show that K_5 and $K_{3,3}$ are non planar.

Or

(b) Prove : $\chi'(K_n) = \begin{cases} n, & \text{if } n > 1 \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$

20. (a) State and prove any three properties of $f(G, \lambda)$.

Or

(b) Derive a necessary and sufficient condition for assigning orientation to the edges of a connected graph so that the resulting digraph is strongly connected.

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