(7 pages) **Reg. No. :** ....

Code No. : 9070 **Sub. Code : PMAM 12**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2017.

First Semester

Mathematics

ANALYSIS — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The Interval $[0,1]$ is

   (a) countable (b) uncountable

   (c) bounded (d) cannot be defined

2. Every finite set is

   (a) open (b) closed

   (c) perfect (d) none

3. If $P>0$ then $\lim\limits_{n\to\infty}\frac{1}{n^P}$ is

   (a) 0 (b) 1

   (c) ∞ (d) none

4. $s_n=\dfrac{(-1)^n}{1+\frac{1}{n}}$ then $\liminf\limits_{n\to\infty} s_n$ is

   (a) 1 (b) -1

   (c) 0 (d) ∞

5. The radius of convergence for the series $\sum n^n 2^n$ is

   (a) ∞ (b) 0

   (c) 1 (d) none

6. The series $\sum\dfrac{\sqrt{n+1}-\sqrt{n}}{n}$

   (a) converges (b) diverges

   (c) oscillates (d) none

7. Monotonic functions have ———— discontinuities of the second kind.

   (a) no (b) only one

   (c) many (d) infinite number

8. The function $f(x)=\begin{cases}1 & x \text{ rational}\\ 0 & x \text{ irrational}\end{cases}$ then f is

(a) continuous

(b) discontinuous of first kind

(c) discontinuous of second kind

(d) none

9. $f(x)=\begin{cases}x^2\sin\frac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$ then $f'(0)$ is

(a) 1 (b) 0

(c) -1 (d) does not exist

10. Let f be defined for all real x and suppose $|f(x)-f(y)|\leq(x-y)^2$ for all real x and y then f is

(a) monotonically increasing

(b) monotonically decreasing

(c) constant

(d) none

PART B — ($5\times5=25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define a neighbourhood of a point p. Prove that every neighbourhood is an open set.

Or

(b) Prove that closed subsets of compact sets are compact.

12. (a) Prove that every convergence sequence in a metric space is bounded.

Or

(b) Suppose $\{s_n\}$ is a complex sequence and $\lim\limits_{n\to\infty}s_n=s$ then prove that $\lim\limits_{n\to\infty}\frac{1}{s_n}=\frac{1}{s}$ provided $s_n\neq0$ $(n=1,2,3..)$ and $s\neq0$.

13. (a) State and prove root test.

Or

(b) For any sequence $\{s_n\}$ of positive numbers prove that $\lim\limits_{n\to\infty} Inf\frac{c_{n+1}}{c_n}\leq\lim\limits_{n\to\infty} Inf\sqrt[n]{c_n}$.

14. (a) If f is a continuous mapping of a metric space X into a metric space Y. Then prove that $f(\overline{E}) \subset \overline{f(E)}$ for every set $E \subset X$. Show by an example that $f(\overline{E})$ can be a proper subset of $\overline{f(E)}$.

Or

(b) Prove that a mapping f of a metric space X into a metric space Y is continuous if and only if $f^{-1}(C)$ is closed in X for every closed set C in Y.

15. (a) State and prove mean value theorem.

Or

(b) Derive the chain rule for differentiation.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Heine Borel theorem.

Or

(b) Prove that a subset E of the real line R' is connected if and only if it has the following property. If $x \in E$, $y \in E$ and $x < z < y$ then $z \in E$.

17. (a) (i) Prove that in any metric space X, every convergent sequence is a cauchy sequence.

(ii) Prove that in R^k, every cauchy sequence converges.

Or

(b) (i) Define a number e, prove that e is irrational.

(ii) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

18. (a) State and prove Merten's theorem.

Or

(b) Suppose $a_n > 0$, $s_n = a_1 + a_2 + ... + a_n$ and $\sum a_n$ diverges.

(i) Prove that $\sum \frac{a_n}{1+a_n}$ diverges.

(ii) Prove that $\frac{a_{N+1}}{s_{N+1}} + \cdots + \frac{a_{N+k}}{s_{N+k}} \geq 1 - \frac{s_n}{s_{N+k}}$ and deduce that $\sum \frac{a_n}{s_n}$ diverges.

19. (a) Let f be a continuous mapping of a compact metric space x into a metric space Y. Then prove that f is uniformly continuous on x.

Or

(b) Let f be monotonically increasing on (a,b). Then prove that $f(x+)$ and $f(x-)$ exist at every point of x of (a,b). more precisely $\sup_{a<t<x} f(t) = f(x-) \leq f(x) \leq f(x+) = \inf_{x<t<b} f(t)$.
Further more if $a < x < y < b$, then prove that $f(x+) \leq f(y-)$.

20. (a) State and prove L' Hospitals Rule.

Or

(b) State and prove Taylor's theorem.
