Reg. No. : ....................................

**Code No. : 7752** **Sub. Code : WMAM 12/ VMAC 12**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

First Semester

Mathematics — Core

REAL ANALYSIS – I

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. Absolute convergence of $\sum a_n$ implies ________

   (a) Divergence

   (b) Convergence

   (c) $|a_n|$ diverges

   (d) All

24. (a) State and prove Merten's theorem.

Or

(b) Assume f has a continuous derivative of order $n+1$ in some open interval I containing c. Define $E_n(x)$ for $x$ in I by $f(x)=\sum \frac{f^{(k)}(c)}{k!}(x-c)^k + E_n(x)$. Then prove that $E_n(x)=\frac{1}{n!}\int_c^x (x-t)^n f^{(n+1)}(t)\,dt$.

25. (a) State and prove the theorem on Cauchy condition for uniform convergence.

Or

(b) Discuss and prove three examples of sequences of real valued functions.

————————

2. The total variation $V_f(a,b)=0$ if and only if $f$ is ________ on $[a,b]$

(a) Continuous

(b) Constant

(c) Variable

(d) $\infty$

3. A series $\sum a_n$ is conditionaly convergent if $\sum a_n$ converges but $\sum |a_n|$ ________

(a) Converges

(b) Conditionaly converges

(c) Diverges

(d) Both (a) and (b)

4. If f and g belongs to $R(\alpha)$ where $\alpha \nearrow$ on $[a,b]$ then the product $fg$ ________

(a) Does not belong to $R(\alpha)$

(b) Belongs to $R(\alpha)$

(c) Both (a) and (b)

(d) None



5. The length of the largest subinterval of the partition $p$ is called as ________

(a) modulus of $p$

(b) norm of $p$

(c) absolute value of $p$

(d) All

6. If $a<b$, then $\int_a^b f\,dx =$ ________ when ever $\int_a^b f\,dx$ exists.

(a) $-\int_a^b f\,dx$ (b) $\int_a^b f\,dx$

(c) 1 (d) $-\int_b^a f\,dx$

7. If $\alpha$ be continuous and $f \nearrow$ on $[a,b]$, then there exists a point $x_0$ in $[a,b]$ such that $\int_a^b f(x)\,d\alpha(x) = f(a)\int_a^{x_0} d\alpha(x) +$ ________

(a) $\int_{x_0}^b d\alpha(x)$ (b) $f(b)\int_{x_0}^b d\alpha(x)$

(c) $f(a)\int_{x_0}^b d\alpha(x)$ (d) $\int_b^{x_0} d\alpha(x)$

8. One of the sufficient condition for the existence of the Riemann integral $\int_a^b f(x)dx$ is ———— on $[a,b]$.

(a) $f$ is continuous (b) $f$ is not continuous

(c) $f$ is compact (d) All

9. If $f \in R$ and $\alpha$ a continuous function on $[a,b]$ whose derivative $\alpha'$ is Riemann integral on $[a,b]$ then $\int_a^b f(x)d\alpha(x)$ ———— $\int_a^b f(x)\, \alpha'(x)\, dx$.

(a) = (b) ≠

(c) < (d) >

10. A function f whose domain is $z^+ \times z^+$ is called a ———— sequence.

(a) Convergent (b) Cauchy

(c) Double (d) Single

11. The double series is said to ———— to the sum a if $\lim_{p,q\to\infty} (S(pq)) = a$.

(a) Converge (b) Diverge

(c) Oscillate (d) All



12. An infinite series of the form $a_0 + \sum_{n=1}^{\infty} a_n(z-z_0)^n$ is called ————

(a) Exponential series

(b) Power series in $(z-z_0)$

(c) Trigonometric series in $(z-z_0)$

(d) Logarithmic series

13. If C is the accumulation point of S, then $\lim_{x\to c}\lim_{n\to\infty} f_n(x) =$ ————

(a) 0

(b) 1

(c) $\lim_{n\to\infty}\lim_{x\to c} f_n(x)$

(d) $\lim_{n\to\infty} f_n(c)$

14. A sequence $\{f_n\}$ is said to be ———— on S if there exists a constant $M > 0$ such that $|f_n(x) \le M|$ for all $x$ in S and all $n$

(a) Uniformly bounded

(b) Converge uniformly

(c) Both (a) and (b)

(d) Diverge

15. Let $\sum f_n(x) = f(x)$ if each $f_n$ is continuous at a point $x_0$ of S, then f is ———— at $x_0$.

(a) Not continuous

(b) Continuous

(c) Diverge

(d) Oscillate

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let f be of bounded variation on $[a, b]$ Let $V(x) = V_f(a, x)$ if $a < x \le b, v(a) = 0$ on $[a, b]$. Then prove the following.

(i) V is an increasing function on $[a, b]$

(ii) V− f is an increasing function on $[a, b]$

Or

(b) If f is continuous on $[a, b]$ and if $f'$ exists and is bounded in the interior, say $|f'(x)| \le A$ for all $x$ in $(a, b)$ then prove that f is of bounded variation on $[a, b]$.



17. (a) Assume that $c \in (a, b)$ if two of the three integrals given below exist prove that the third also exists and we have

$$\int_a^c f\, d\alpha + \int_c^b f\, d\alpha = \int_a^b f\, d\alpha .$$

Or

(b) Let $\alpha \nearrow$ on $[a, b]$ if $f \in R(\alpha)$ on $[a, b]$ then show that $|f| \in R(\alpha)$ on $[a, b]$ and

$$\left|\int_a^b f(x)\, d\alpha(x)\right| \le \int_a^b |f(x)|\, d\alpha(x) .$$

18. (a) State and prove second fundamental theorem of integral calculus.

Or

(b) Assume that $\alpha$ is continuous and that $f \nearrow$ on $[a, b]$ then prove that there exists a point $x_0$ in $[a, b]$ such that

$$\int_a^b f(x)\, d\alpha(x) = f(a)\int_a^{x_0} d\alpha(x) + f(b)\int_{x_0}^b d\alpha(x) .$$

19. (a) Let $a_n > 0$ then show that the product $\pi(1+a_n)$ converges iff the series $\sum a_n$ converges.

Or

(b) Show that if a series is convergent with sum S, then it is also $(C,1)$ summable with cesaro sum S.

20. (a) Assume that $f_n \to f$ uniformly on S. If each $f_n$ is continuous at a point c of s, then prove that the limit function f is also continuous at C .

Or

(b) State and prove Dirichlet's test for uniform convergence.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) State and prove Riemann's theorem on conditionally convergent series.

Or

(b) Let F be of bounded variation on $[a, b]$ and assume that $C \in (a,b)$ then prove that f is of bounded variation on $[a, c]$ and on $[c, b]$ and $V_f(a, b) = V_f(a, c) + V_f(c,b)$.



22. (a) Assume that $\alpha \nearrow$ on $[a,b]$. The prove that the following statement are equivalent.

(i) $f \in R(\alpha)$ on $[a,b]$

(ii) f satisfies riemann's condition with respect n ot $\alpha$ on $[a,b]$

(iii) $\underline{I}(f, \alpha) = \overline{I}(f,\alpha)$.

Or

(b) If $f \in R(\alpha)$ on $[a,b]$ then prove that $\alpha \in R(f)$ on $[a,b]$ and

$$\int_a^b f(x)\,d\alpha(x) + \int_a^b \alpha(x)\,df(x) = f(b)\alpha(b) - f(a)\alpha(a)$$

23. (a) Assume that $\alpha$ is of bounded variation on $[a,b]$. Let $V(x)$ denote the total variation of $\alpha$ on $[a,x]$ if $a < x \le b$ and $v(a) = 0$ Let f be defined and bounded on $[a,b]$ If $f \in R(\alpha)$ on $[a,b]$ then prove that $f \in R(V)$ on $[a,b]$.

Or

(b) Discuss the theorem on change of variable in a Riemann integral.