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Reg. No. : .....

Code No. : 6384 **Sub. Code : ZPHM 12**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

First Semester

Physics – Core

MATHEMATICAL PHYSICS – I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The divergence of a vector field is always

   (a) a vector

   (b) a scalar

   (c) some times a scalar and sometimes a vector

   (d) neither a scalar nor a vector

2. If $\varphi = 4e^{(2x-y+z)}$, then grad φ at $(1, 1, -1)$ is

   (a) $4\left(2\hat{i} - \hat{j} + \hat{k}\right)$

   (b) $2\hat{i} - \hat{j} + \hat{k}$

   (c) $8\hat{i} - \hat{j} + 2\hat{k}$

   (d) $3\hat{i} + 6\hat{j} + 9\hat{k}$

3. Which of the following is the Laguerre's equation?

   (a) $\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - ny = 0$

   (b) $\frac{d^2y}{dx^2} - ny = 0$

   (c) $x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ny = 0$

   (d) $x\frac{d^2y}{dx^2} + ny = 0$

4. The value of $P_1(x)$ is

   (a) 1 (b) 0

   (c) x^2 (d) x

5. Heat flow equation is

(a) $\nabla^2\varphi = 0$

(b) $\nabla^2\varphi = \frac{1}{h^2}\frac{\partial\varphi}{\partial t}$

(c) $\nabla^2\varphi = 1$

(d) $\nabla^2\varphi = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$

6. Which of the following represents wave equation for the membrane?

(a) $\frac{\partial^2 u}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$

(b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$

(c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$

(d) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$

7. If A_{ij} is anti symmetric tensor, then the component A_{11} is

(a) 1 (b) -1

(c) 2 (d) 0

8. The divergence of a contra variant vector A^{μ} is

(a) $A;^{\mu}_{\mu}$ (b) $A;\mu$

(c) A (d) $\nabla\varphi$

9. From the pack of a 52 cards, one is drawn at random, the probability of getting a king is

(a) $\frac{2}{13}$ (b) $\frac{1}{169}$

(c) $\frac{25}{169}$ (d) $\frac{1}{13}$

10. The value of $\Sigma f\left(x-\bar{x}\right)$ is

(a) negative (b) positive

(c) 0 (d) arbitrary

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Check whether the following vector are linearly dependent or independent $(1, 2, -3)$, $(2, 5, 1)$, $(-1, 1, 4)$.

Or

(b) Find a unit vector perpendicular to the surface $x^2 + y^2 - z^2 = 11$ at the point $(4, 2, 3)$.

12. (a) Calculate the Wronskian of the functions $e^{(p-iq)x}$ and $e^{(p+iq)x}$.

Or

(b) State and prove the generating function for Laguerre polynomials.

13. (a) Solve the heat flow equation.

Or

(b) A string of length l fixed at both ends is plucked at a distance d from one fixed point by an amount h. Find the displacement at any position at any instant of time.

14. (a) Show that the transformations of tensors form a group.

Or

(b) Evaluate $\nabla \times \nabla \phi$.

15. (a) State and prove additive law of probability.

Or

(b) The following data are the number of seeds germinating out of 10 on damp filter for 80 sets of seeds. Fit a Binomial distribution to these data:

x :	0	1	2	3	4	5	6	7	8	9	10
f :	6	20	23	12	8	6	0	0	0	0	0

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Verify divergence theorem for the vector $A = x^2 \hat{i} + y^2 \hat{j} + z^2 \hat{k}$ taken over the cube $0 \le x, y, z \le 1$.

Or

(b) State and prove Stoke's theorem.

17. (a) Solve $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$.

Or

(b) Show that

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = 0 \text{ for } m \neq n$$

$$\int_{-1}^{1} [P_n(x)]^2\, dx = \frac{2}{2n+1}.$$

18. (a) Solve the differential equation

$$2x\frac{\partial u}{\partial x} - 3y\frac{\partial u}{\partial y} = 0.$$

Or

(b) A string of length l with fixed ends is plucked up at its centre a distance h from the position of equilibrium and then released. Find

(i) the displacement at any position at any time

(ii) the normal frequencies and normal modes for the vibrating string.

19. (a) A covariant tensor has components xy, $2y - x^2$, xz in rectangular coordinates. Find its covariant components in spherical coordinates.

Or

(b) Derive Riemann – Christoffel tensor.

20. (a) Calculate:

(i) the quartile

(ii) the mean and

(iii) the standard deviation wages from the following data:

Weakly wages in dollars :	35-36	36-37	37-38	38-39	39-40	40-41	41-42
No. of wage earners:	14	20	42	54	45	18	7

Or

(b) Derive the normal distribution as the limiting case of binomial distribution when $p = q$.