(7 pages) **Reg. No. :** .....

Code No. : 30732 E **Sub. Code : EEMA 11/ FEMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024

First Semester

Mathematics

Elective — ALGEBRA AND DIFFERENTIAL EQUATION

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL the questions.

Choose the correct answer.

1. The smallest degree of an equation with rational coefficients, two of whose roots are $2+3i$ and $2-3i$ roots is _____.

   (a) 2 (b) 4

   (c) 6 (d) 3

2. The sum of the squares of the roots of $x^3 + ax^2 - bx + c = 0$ is _____.

   (a) $a^2 - 2b$ (b) $a^2 + 2b$

   (c) $b^2 - 2c$ (d) $a^2 + 2c$

3. If the roots of $x^3 - 8x^2 + 19x - 12 = 0$ are 1, 3, 4 then the roots of $x^3 - 16x^2 + 76x - 96 = 0$ are _____.

   (a) 1, 3, 4 (b) –1, –3, –4

   (c) 2, 5, 8 (d) 1, 9, 16

4. Horner's method and Newton's method are used to find _____.

   (a) the exact values of the roots of quadratic equations

   (b) approximate values of the real roots of an equation

   (c) approximate values of the complex roots of quadratic equations

   (d) the positive real roots of an equation

5. If $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = 0$ then x is _____.

   (a) $\frac{1}{2}$ (b) 1

   (c) ± 1 (d) 0

6. The eigen values of $\begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix}$ are

(a) 2, 2 (b) 2, 3

(c) 2, 4 (d) 3, 4

7. The solution of $\frac{dy}{dx} = \frac{-y}{x}$ is

(a) $xy = c$ (b) $cx = y$

(c) $c = x^2 y$ (d) $x = cy$

8. The solution of $y = (x-a)p - p^2$ is _____.

(a) $(x-a)c - c^2$ (b) $(x-c)a - c^2$

(c) $(x-a)c = c^2$ (d) $(x-a)a = c^2$

9. $L(t^n) =$ _____.

(a) $\frac{n!}{S^{n+1}}$ (b) $\frac{n!}{S^{t+1}}$

(c) $\frac{t!}{S^{n+1}}$ (d) $\frac{t!}{S^{t+1}}$

10. $L^{-1}(\sin at) =$ _____.

(a) $\frac{a}{s^2 - a^2}$ (b) $\frac{s}{s^2 - a^2}$

(c) $\frac{a}{s^2 + a^2}$ (d) $\frac{s}{s^2 + a^2}$

PART B — (5 × 5 = 25 marks)

Answer ALL the questions choosing either (a) or (b).

11. (a) Solve $x^4 - 4x^2 + 8x + 35 = 0$ given that $2 + \sqrt{2i}$ is a root.

Or

(b) If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ find $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$.

12. (a) Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2.

Or

(b) Find the negative root of $x^3 - 2x + 5 = 0$ correct to two places of decimals.

13. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ hence find A^{-1}.

Or

(b) Find the sum and product of the eigen values of the matrix $\begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}$ without actually finding the eigen values.

14. (a) Solve $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$.

Or

(b) Solve $y = px + \frac{a}{p}$.

15. (a) Find $L(\sin^2 2t)$.

Or

(b) Find $L^{-1}\left(\frac{s}{(s^2 + a^2)^2}\right)$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL the questions choosing either (a) or (b).

16. (a) Solve the equation

$8x^4 - 90x^3 + 315x^2 - 405x + 162 = 0$ given that the roots are in G.P.

Or

(b) Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

17. (a) Find Newton's method the root of the equation

(i) $x^3 - 2x - 2 = 0$ which is nearer to 2.

(ii) $x^3 - 2x - 5 = 0$ which lies between 2 and 3 correct to 2 places of decimals.

Or

(b) Apply Horner's method to find the root of the equation $x^3 - 9x^2 + 23x - 14 = 0$ which lies between 4 and 5 correct to 2 places of decimals.

18. (a) Find the inverse of the matrix $\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ using Cayley-Hamilton theorem.

Or

(b) Prove that the product of the eigen values is $|A|$.

19. (a) Solve $xp(3y^2 - ax) = y(2y^2 - ax)$.

Or

(b) Solve $(xp - y)^2 = a(1 + p^2)(x^2 + y^2)^{\frac{3}{2}}$.

20. (a) Find $L^{-1}\left(\frac{s^2-s+2}{s(s-3)(s+2)}\right)$.

Or

(b) Solve the equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$ given that $y=\frac{dy}{dt}=0$ when $t=0$.
