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Reg. No. :

Code No. : 30937 E Sub. Code : FCMA 12

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2024.

First Semester

Mathematics-Core

DIFFERENTIAL CALCULUS

(For those who joined in July 2024 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. $\frac{d}{dx} (\cos h^{-1} x) = \underline{\hspace{2cm}}$

(a) $\frac{1}{\sqrt{x^2 - 1}}$

(b) $\frac{-1}{\sqrt{x^2 - 1}}$

(c) $\frac{1}{\sqrt{x^2 + 1}}$

(d) $\frac{-1}{\sqrt{x^2 + 1}}$

2. $\frac{d^5}{dx^5} (e^{2x})$ is _____.

(a) $16e^{2x}$

(b) $32e^{2x}$

(c) $2^4 e^{2x}$

(d) None

3. $\frac{\partial^2 f}{\partial x \partial y}$ is denoted by.

(a) f_{xy}

(b) f_{yx}

(c) f_{xx}

(d) f_{yy}

4. If $x = r \cos \theta$, then $\frac{\partial x}{\partial \theta}$ is.

(a) $r \sin \theta$

(b) $r \cos \theta$

(c) $-r \cos \theta$

(d) $-r \sin \theta$

5. $f(x, y, z)$ is a homogenous function of 2nd degree,
 $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} = \underline{\hspace{2cm}}$.

(a) $2f$

(b) $2^2 f$

(c) $2^3 f$

(d) f



6. If $AC - B^2 > 0$ and $A > 0$ (or $B > 0$) then $f(a, b)$ is.

- (a) maximum
- (b) minimum
- (c) neither (a) or (b)
- (d) (a) or (b)

7. The Parametric form of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is.

- (a) $(a \cos \theta, a \sin \theta)$
- (b) $(a \sec \theta, b \tan \theta)$
- (c) $(a \cos \theta, b \sin \theta)$
- (d) $(a \cos \theta, b \cos n \theta)$

8. For $f(x, y, \alpha) = 0$, $\frac{dy}{dx}$ at $(\xi, \eta) =$ _____.

- (a) $\frac{\partial f / \partial \xi}{\partial f / \partial \eta}$
- (b) $\frac{\partial f / \partial \eta}{\partial f / \partial \xi}$
- (c) $\frac{-\partial f / \partial \xi}{\partial f / \partial \eta}$
- (d) $\frac{-\partial f / \partial \eta}{\partial f / \partial \xi}$

9. For a circle, radius of curvature is equal to.

- (a) radius of the circle
- (b) square of the radius
- (c) square root of the radius
- (d) reciprocal of radius

10. On a given curve, at any point curvature is.

- (a) $\frac{ds}{d\psi}$
- (b) $\frac{d\psi}{ds}$
- (c) $\frac{ds}{dx}$
- (d) $\frac{ds}{dy}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that

$$\frac{d^n}{dx^n} \left(\frac{1}{(ax+b)^2} \right) = \frac{(-1)^n a^n (n+1)!}{(ax+b)^{n+2}}.$$

Or

(b) Find y_n , where $y = \frac{3}{(x+1)(2x-1)}$.



12. (a) If $u = \log(\tan x \tan y + \tan z)$ prove

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2.$$

Or

- (b) Find $\frac{du}{dx}$ when $u = x^2 + y^2$ where
 $y = \frac{1-x}{x}$

13. (a) If $u = \sin^{-1} \left[\frac{x^2 + y^2}{x + y} \right]$, prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u.$$

Or

- (b) If $z = f(x, y)$ and $x = r \cos \theta$, $y = r \sin \theta$,
 prove that

$$\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 = \left(\frac{\partial z}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta} \right)^2$$

14. (a) Find the envelope of the family of straight lines $x \cos \alpha + y \sin \alpha = a \sec \alpha$, α being the parameter.

Or

- (b) Find the envelope of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ subject to $a^2 + b^2 + c^2 = C$, where C is constant.

15. (a) Find P at $x = c$ on $xy = c^2$.

Or

- (b) Find the radius of curvature of the curve $r^n = a^n \cos n\theta$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) If $y = (\sin^{-1} x)^2$ prove that

$$(1 - x^2) y_{n+2} - (2n + 1)x y_{n+1} - n^2 y_n = 0.$$

Or

- (b) Find the n^{th} derivative of $x^2 \log 3x$.

17. (a) If $u = \log(x^2 + y^2 + z^2)$, prove

$$x \frac{\partial^2 u}{\partial y \partial z} = y \frac{\partial^2 u}{\partial z \partial x} = z \frac{\partial^2 u}{\partial x \partial y}.$$

Or



(b) If $z = (1 - 2xy + y^2)^{-\frac{1}{2}}$ prove

$$x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = y^2 z^3.$$

18. (a) Find the maximum and minimum of $x^2 + y^2 + 6x + 12$.

Or

- (b) In a plane triangle ABC , find the maximum value of $\cos A \cos B \cos C$.

19. (a) Find the envelope of the family of straight lines $\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$.

Or

- (b) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ subject to $a^n + b^n = c^n$, C is constant.

20. (a) Show that the evolute of the cycloid $X = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ is another cycloid.

Or

- (b) Find the radius of curvature at $(a \cos^3 \theta, a \sin^3 \theta)$ on $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

