(7 pages) **Reg. No. :** ....

Code No. : 5384 **Sub. Code : ZMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2024.

Fourth Semester

Mathematics – Core

COMPLEX ANALYSIS

(For those who joined in July 2021 – 2022)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The geometric series $1+z+z^2+...+z^n+...$ diverges for ————

   (a) $|z| \geq 1$ (b) $|z| < 1$

   (c) $|z| = 0$ (d) $|z\bar{z}| = 1$

2. The sum of the orders of the zeros of a polynomial is equal to its ————

   (a) degree (b) derivative

   (c) integration (d) order

3. The mapping such that two curves which form an angle at z_0 are mapped upon curves forming the same angle in magnitude and direction is called ———— at all points.

   (a) bijective (b) one-one

   (c) conformal (d) onto

4. The cross ratio (z_1, z_2, z_3, z_4) is the image of z_1 under the linear transformation which carries z_2, z_3, z_4 into ————

   (a) 1, 1, 1 (b) 1, 0, ∞

   (c) 1, 0, 1 (d) 1, 0, 0

5. The length of circle $z = z(t) = a + \rho e^{it}$, $0 \leq t \leq 2\pi$ is ————

   (a) 2π (b) $2\pi i$

   (c) $2\pi\rho$ (d) 2ρ

6. The integral $\int_\gamma f\,dz$ with continuous f depends only on the end points of γ if and only if f is the ———— of an analytic function in Ω.

   (a) derivative (b) integrand

   (c) zero (d) discontinuous

7. A function which is analytic and bounded in the whole plane must reduce to ———

(a) constant (b) variable

(c) integer (d) 0

8. If $\lim_{z \to a} f(z) = \infty$ then the point a is called a ——— of $f(z)$.

(a) limit (b) order

(c) pole (d) radius

9. A cycle γ is said to ——— the region Ω if and only if $n(\gamma, a)$ is equal to 1 for all points $a \in \Omega$ and either undefined or 0 for all $a \notin \Omega$

(a) closed (b) bound

(c) open (d) compact

10. The ——— of $f(z)$ at an isolated singularity a is the unique number R such that $f(z) - R/z - a$ the derivative of a single valued analytic function in an annulus $0 < |z - a| < \delta$.

(a) pole (b) zero

(c) order (d) residue

PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Find the conjugate of the harmonic function $u = x^2 - y^2$.

Or

(b) Derive Cauchy – Riemann equations.

12. (a) Reflect the imaginary axis, the line $x = y$ and the circle $|z| = 1$ in the circle $|z - 2| = 1$.

Or

(b) Explain the symmetry principle.

13. (a) Prove that $\int_{-\gamma} f(z)dz = -\int_{\gamma} f(z)dz$.

Or

(b) Let $f(z)$ be analytic on the set R' obtained by omitting a finite number of interior points J_i from a rectangle R. If $\lim_{z \to J_i} (z - J_j) f(z) = 0$ for all j then prove that $\int_{\partial R} f(z)dz = 0$.

14. (a) If the piecewise differentiable closed curve γ does not pass through the point a, then prove that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

Or

(b) Suppose that $f(z)$ is analytic in an open disk Δ, γ be a closed curve in Δ. For any point a not on γ prove that $n(\gamma, a) \cdot f(a) = \frac{1}{2\pi i} \int_{\gamma} \frac{f(z)dz}{z-a}$, where $n(\gamma, a)$ is the index of a with respect to γ.

15. (a) Find the residue of the function $\frac{e^z}{(z-a)(z-b)}$.

Or

(b) State and prove arguement principle.

PART C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) State and prove Abel's theorem.

Or

(b) Derive Taylor – Maclaurin development with the assumption that $f(z)$ has a power series expansion.

17. (a) State and prove the symmetric principle.

Or

(b) Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on straight line.

18. (a) Prove that the line integral $\int_{\gamma} pdx + qdy$ defined in Ω depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p$, $\frac{\partial U}{\partial y} = q$.

Or

(b) State and prove Cauchy's theorem for a rectangle.

19. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

Or

(b) State and prove Taylor's theorem.

20. (a) Derive Cauchy residue theorem.

Or

(b) Compute $\int_0^{\pi} \frac{d\theta}{a + \cos\theta}$, $a > 1$.
