(8 pages)

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B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fifth Semester

Mathematics — Core

TRANSFORMS AND THEIR APPLICATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $F\{f(x)\} = \bar{f}(s)$, then $F\{e^{iax} f(x)\} =$ ________

(a) $\bar{f}(s+a)$ (b) $\bar{f}(x+a)$

(c) $\bar{f}(s-a)$ (d) $\bar{f}(x-a)$

2. $F_S\{f''(x)\} =$ ________

(a) $-sF_s\{f(x)\} + f(0)$

(b) $-s^2 F_s\{f(x)\} + sf(0)$

(c) $-sF_c\{f(x)\} + f(0)$

(d) $-s^2 F_c\{f(x)\} + sf(0)$

3. If $F\{f(x)\} = \bar{f}(s)$, then $F\{f(ax)\} =$ ________

(a) $\frac{1}{|a|}\bar{f}\left(\frac{s}{a}\right)$ (b) $\bar{f}\left(\frac{s}{a}\right)$

(c) $|a|\bar{f}(sa)$ (d) $\bar{f}(sa)$

4. $\frac{d}{ds}\{F_c(f(x))\} =$ ________

(a) $F_c\{xf(x)\}$ (b) $-F_c\{xf(x)\}$

(c) $F_s\{xf(x)\}$ (d) $-F_s\{xf(x)\}$

5. $F_c\{f(x)\} = \bar{f}_c(n) =$ ________

(a) $\int_0^l f(x)\sin\left(\frac{n\pi.x}{l}\right)dx$

(b) $\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$

(c) $\int_0^l f(x)\sin(n\pi x)dx$

(d) $\int_0^l f(x)\cos(n\pi x)dx$

6. $F_c\{f'(x)\} =$ ________

(a) $\frac{-n\pi}{l}\bar{f}_c(n)$

(b) $(-1)^n f(l) - f(0) + \frac{n\pi}{l}\bar{f}_s(n)$

(c) $\frac{n\pi}{l} f_c(n)$

(d) $(-1)^n f(l) + f(0) - \frac{n\pi}{l}\bar{f}_s(n)$

7. $Z(n) =$ ________, where ROC is $|z| > 1$.

(a) $\frac{z}{(z+1)^2}$ (b) $\frac{z(z+1)}{(z-1)^3}$

(c) $\frac{z}{(z-1)^2}$ (d) $\frac{z(z-1)}{(z+1)^2}$

8. $z(a^n) =$ ________, if $|z| > a$.

(a) $\frac{z}{z-a}$ (b) $\frac{z}{z+a}$

(c) $\frac{nz}{z-a}$ (d) $\frac{nz}{z+a}$

9. $z^{-1}\left\{\frac{1}{z+2}\right\} =$ ________

(a) 2^n (b) $(-2)^n$

(c) $(-2)^{n-1}$ (d) 2^{n-1}

10. $z^{-1}\left\{e^{a/z}\right\} =$ ________

(a) a^{n-1} (b) a^n

(c) $\frac{a^{n-1}}{(n-1)!}$ (d) $\frac{a_n}{n!}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the Fourier transform of $f(x)$, defined as $f(x) = \begin{cases} 1, & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases}$ and hence find the value of $\int_0^\infty \frac{\sin x}{x} dx$.

Or

(b) Find the Fourier transform of $\left\{\frac{\sin ax}{x}\right\}$ and hence prove that $\int_{-\infty}^{\infty} \frac{\sin^2 ax}{x^2} = a\pi$.

12. (a) Find the Fourier cosine transform of e^{-ax} and use it to find the Fourier transform of $e^{-a|x|}\cos bx$.

Or

(b) Find $F_c(e^{-a^2x^2})$.

13. (a) Find the finite Fourier sine and cosine transforms of $\left[1-\frac{x}{\pi}\right]^2$ in $(0, \pi)$.

Or

(b) Find the finite Fourier sine and cosine transforms of e^{ax} in $(0, l)$.

14. (a) Find the z – transform of t^2e^{-t}.

Or

(b) Find the z – transform of $n\cos n\theta$.

15. (a) Find $z^{-1}\left\{\frac{1}{1+4z^{-2}}\right\}$ by the long division method.

Or

(b) Find $z^{-1}\left\{\frac{2z^2+4z}{(z-2)^3}\right\}$, by using Residue theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$, subject to the initial conditions $y(x,0) = f(x)$, $-\infty < x < \infty$. $\frac{\partial y}{\partial t}(x, 0) = g(x)$ and the boundary conditions $y(x, t) \to 0$ as $x \to \pm\infty$.

Or

(b) Solve the equation $(D^2 - 4D + 3)y = \cos 3x$, $x > 0$, given that $y(0) = 0$ and $y'(0) = 0$.

17. (a) Find $f(x)$, if its Fourier sine transform is $\left(\frac{s}{s^2+1}\right)$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}$, satisfying the boundary conditions $\frac{\partial u}{\partial x}(0, t) = k$, $t \geq 0$ and $u(x,t) \to 0$ as $x \to 0$ and the initial condition $u(x, 0) = 0$.

18. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$, using finite Fourier transforms, given that $u(0,t) = 0$, $u(\pi, t) = 0$, for $t > 0$ and $u(x, 0) = 4\sin^3 x$.

Or

(b) Solve the equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < l$, using finite Fourier transform, given that $\frac{\partial u}{\partial x}(0, t) = 0$, $\frac{\partial u}{\partial x}(l, t) = 0$ for $t > 0$ and $u(x,0) = kx$, for $0 < x < l$.

19. (a) Find the $z-$transform of the following functions

(i) $r^n \cos n\theta$

(ii) $r^n \sin n\theta$

(iii) $\cos n\theta$

(iv) $\sin n\theta$.

Or

(b) Find $z-$transform of

(i) $f(n) = \frac{1}{n(n-1)}$ and

(ii) $f(n) = \frac{2n+3}{(n+1)(n+2)}$.

20. (a) Find $z^{-1}\left\{\frac{z^2+2z}{z^2+2z+4}\right\}$ by the method of partial fractions.

Or

(b) Solve the simultaneous difference equations.

$x_{n+1} = 7x_n + 10y_n$; $y_{n+1} = x_n + 4y_n$, given that $x_0 = 3$ and $y_0 = 2$.
