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M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2013.

Third Semester

Mathematics

COMPLEX ANALYSIS

(For those who joined in July 2012 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

1. The quotient $\frac{f(x)}{g(x)}$ of two continuous functions $f(x)$ and $g(x)$ is defined and continuous at 'a' if _____
- (a) $g(x) \neq 0$ (b) $g(a) \neq 0$
(c) $f(x) \neq 0$ (d) $f(a) \neq 0$
2. A function u is said to be harmonic if it satisfies _____
- (a) $\Delta u \neq 0$ (b) $\Delta u = 0$
(c) $u = 0$ (d) None of these

3. $x, e^x > 0$ if
- (a) x is real (b) $x < 0$
(c) $x > 0$ (d) none of these
4. The value of $\sin i$ is _____
- (a) $\frac{1}{e}$ (b) e
(c) 1 (d) none of these
5. With respect to an orientation of a circle C determined by an ordered triple of points z_1, z_2, z_3 on C , a point z not on C is said to lie to the right of C if $Im(z, z_1, z_2, z_3) \dots$
- (a) $= 0$ (b) < 0
(c) > 0 (d) none of these
6. A linear transformation with coinciding fixed points is said to be _____
- (a) hyperbolic (b) elliptic
(c) parabolic (d) loxodromic
7. With usual notation, $n(-\gamma, a) =$
- (a) $n(\gamma, a)$ (b) $-n(\gamma, a)$
(c) $\frac{1}{n(\gamma, a)}$ (d) none of these



8. If γ lies inside of a circle, then $n(\gamma, a) =$ _____ for all points 'a' outside of the same circle.
- (a) 1 (b) 2
(c) 0 (d) 3
9. The founder of complex integration theory is _____
- (a) Riemann (b) Cauchy
(c) Weierstrass (d) Taylor
10. A cycle γ is said to bound the region Ω if and only if $n(\gamma, a)$ is defined and equal to _____ for all points $a \in \Omega$
- (a) 0 (b) 1
(c) 2 (d) none of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Prove that the functions $f(z)$ and $\overline{f(\overline{z})}$ are analytic simultaneously.
- Or
- (b) If $g(w)$ and $f(z)$ are analytic functions, show that $g(f(z))$ is also analytic.

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12. (a) Show that the exponential function e^z satisfies the addition theorem.

Or

- (b) Show that
- (i) $\log(z_1 z_2) = \log(z_1) + \log(z_2)$
(ii) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

13. (a) Find the fixed points of the linear transformation $w = \frac{z}{2z-1}$. Is the transformation elliptic, hyperbolic, or parabolic?

Or

- (b) Find the fixed points of the linear transformation $w = \frac{3z-4}{z-1}$. Is the transformation elliptic, hyperbolic, or parabolic?

14. (a) Compute $\int_{|z|=2} \frac{dz}{z^2+1}$.

Or

- (b) Compute $\int_{|z|=2} \frac{dz}{z^{2+1}}$.

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[P.T.O.]



15. (a) If $f(z)$ is analytic and non-constant in a region Ω , then prove that its absolute value $|f(z)|$ has no maximum in Ω .

Or

- (b) If $f(z)$ is defined and continuous on a closed bounded set E and analytic on the interior of E , then prove that the maximum of $|f(z)|$ on E is assumed on the boundary of E .

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) If $u(x, y)$ and $v(x, y)$ have continuous first-order partial derivatives which satisfy the Cauchy-Riemann differential equations, then prove that $f(z) = u(z) + iv(z)$ is analytic with continuous derivative $f'(z)$, and conversely.

Or

- (b) If all zeros of a polynomial $P(z)$ lie in a half plane, then prove that all zeros of the derivative $P'(z)$ lie in the same half plane.

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17. (a) Show that any linear transformation which transforms the real axis into itself can be written with real coefficients.

Or

- (b) Find the linear transformation which carries the circle $|z| = 2$ into $|z + 1| = 1$, the point -2 into the origin, and the origin into i .

18. (a) Find all the circles which are orthogonal to $|z| = 1$ and $|z - 1| = 4$.

Or

- (b) Prove that the line integral $\int_{\gamma} p dx + q dy$, defined in Ω , depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q$.

19. (a) State and prove Cauchy's integral formula.

Or

- (b) Show that an analytic function has derivatives of all orders, which are then also analytic.

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20. (a) State and prove Schwarz's lemma.

Or

(b) Show that $|f(z)| \leq 1$ for $|z| \leq 1$ implies

$$\frac{|f'(z)|}{(1 - |f(z)|^2)} \leq \frac{1}{1 - |z|^2}.$$

