(7 pages) **Reg. No. :** ....

Code No. : 20430 E **Sub. Code : CSMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Third Semester

Mathematics

Skill Based Subject – VECTOR CALCULUS

(For those who joined in July 2021 – 2022)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

Choose the correct answer :

1. If $\vec{a}, \vec{b}$ are functions of a scalar variable u then $\frac{d}{du}(\vec{a}\cdot\vec{b}) =$ ____________.

(a) $\frac{d\vec{a}}{du}\cdot\frac{d\vec{b}}{du}$ (b) $\frac{d\vec{a}}{du}\cdot\vec{b}+\vec{a}\cdot\frac{d\vec{b}}{du}$

(c) $\frac{d\vec{a}}{du}+\frac{d\vec{b}}{du}$ (d) 0

2. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $|\vec{r}| = r$ then ∇r is

(a) 0 (b) r

(c) $\vec{r}$ (d) $\frac{\vec{r}}{r}$

3. If ϕ,ψ are scalar point functions then $\nabla(\phi\psi) =$ ____________.

(a) $\phi\nabla\psi + \psi\nabla\phi$ (b) $\nabla\phi\nabla\psi$

(c) 0 (d) $\phi'\psi'$

4. If $\nabla\times\vec{V} = 0$, $\vec{V}$ is said to be ____________ vector.

(a) irrotational (b) solenoidal

(c) variable (d) constant

5. If a vector field $\vec{F}$ is such that $\vec{f} = \nabla\phi$ then $\vec{f}$ is said to be

(a) variable (b) conservative field

(c) parallel field (d) none

6. If $\vec{f} = 2x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla\circ\vec{f} =$ ____________.

(a) 0 (b) 2

(c) 1 (d) 4

7. If $\vec{f} = x\vec{i} + y\vec{j} + az\vec{k}$ is solenoidal then a is ____________.

(a) 0 (b) 2

(c) –2 (d) 1

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8. The necessary and sufficient condition that $\int \vec{f} \circ d\vec{r}$ be independent of the path is

(a) $\vec{f} = \nabla\phi$ (b) $\nabla \circ \vec{f} = 0$

(c) $\vec{f} = \frac{\nabla\phi}{|\nabla\phi|}$ (d) $\nabla \times \vec{f} = 0$

9. By stoke's theorem $\iint_s (\nabla \times \vec{f}) \cdot \vec{n} ds =$ ____________.

(a) 0 (b) $\int_C \vec{f} \circ d\vec{r}$

(c) $\int_C \vec{r} \circ d\vec{f}$ (d) 1

10. $\iint_s (\nabla \times \vec{f}) \cdot \vec{n} ds$ ____________.

(a) V (b) 2V

(c) 3V (d) 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\vec{A}, \vec{B}, \vec{C}$ are functions of the scalar variable u, derive an expression for $\frac{d}{du}\left(\vec{A} \times \left(\vec{B} \times \vec{C}\right)\right)$.

Or

(b) (i) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $|\vec{r}| = r$ then show that $\nabla(r^n) = nr^{n-2}\vec{r}$.

(ii) If $\nabla\phi = 5r^3\vec{r}$, find ϕ.

12. (a) If C is the circle $x = 3\cos t$, $y = 3\sin t$, $z = 0$ and $\vec{f} = (2x - y + z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}$ then find the value of $\int_C \vec{f} \circ d\vec{r}$.

Or

(b) Prove that $\nabla\left(\frac{\phi}{\psi}\right) = \frac{\phi\nabla\psi - \psi\nabla\phi}{\psi^2}$.

13. (a) If $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational, find the value of a, b, c.

Or

(b) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $P(1, -2, -1)$ in the direction of PQ where Q is $(3, -3, -2)$.

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14\. (a) If C is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z=0$ then find the value of $\oint_C \left(\frac{b^2x^2}{a^2}+\frac{a^2y^2}{b^2}\right)^{\frac{1}{2}} ds$.

Or

(b) If S is the surface of the sphere $x^2+y^2+z^2=1$, evaluate $\iint_S \left(x\vec{i}+2y\vec{j}+3z\vec{k}\right)\cdot ds$.

15\. (a) Evaluate $\iiint_V \nabla\cdot\vec{F}dV$ where $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ and V is the volume enclosed by the cube $0\le x,y,z\le 1$.

Or

(b) Evaluate $\int_C (e^x dx+2ydy-dz)$ by stoke's theorem where C is the curve $x^2+y^2=4$, $z=2$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16\. (a) Prove that $\nabla^2 f(r)=\frac{\partial^2 f}{\partial r^2}+\frac{2}{r}\frac{\partial f}{\partial r}$.

Or

(b) $\vec{u},\vec{v},\vec{w}$ are three mutually perpendicular unit vectors whose directions vary with a scalar variable t. Show that $\vec{u}',\vec{v}'\vec{w}'$ are coplanar where the differentiation is with respect to t.

17\. (a) (i) Prove that $\nabla\cdot(\nabla\times\vec{A})=\nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A}$.

(ii) If $\nabla\phi=(6xy+z^3)\vec{i}+(3x^2-z)\vec{j}+(3xz^2-y)\vec{k}$, find ϕ.

Or

(b) Find the directional derivative of $f(x,y,z)=x^2yz+4xz^2$ at the point $(1,-2,-1)$ in the direction of $2\vec{i}-\vec{j}-2\vec{k}$.

18\. (a) Show that the integral of $\vec{F}=(3x^2+6xy)\vec{i}+(3x^2-y^2)\vec{j}$ is independent of the path of integration. Find $\int_C \vec{F}\cdot d\vec{r}$ along the curve joining (0, 0) and (1, 2).

Or

(b) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F}=xy\vec{i}+(x^2+y^2)\vec{j}$ and C is the rectangle in the xy-plane bounded by the lines $y=2$, $x=4$, $y=10$, $x=1$.

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19. (a) Verify Green's theorem for $\int_C (x-2y)dx + xdy$ where C is the circle $x^2 + y^2 = 1$.

Or

(b) Evaluate $\iint_S \vec{A} \circ \vec{n}\, dS$ if $\vec{A} = yz\vec{i} + 2y^2\vec{j} + xz^2\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 1$ contained in the first octant wbetween the planes $z = 0$ and $z = 2$.

20. (a) Verify stoke's theorem where $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$ where S is the surface of the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$.

Or

(b) Verify Gauss divergence theorem for the vector function $\vec{F} = 2xz\vec{i} + yz\vec{j} + z^2\vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

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