(7 pages) **Reg. No. : ....**

Code No. : 6853 **Sub. Code : PMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Fourth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $u+iv=z^3$, then $\frac{\partial u}{\partial x}$ is

   (a) $3x^2-3y^2$ (b) $3x^2+3y^2$

   (c) $6xy$ (d) $-6xy$

2. A function u which satisfies $\Delta u=0$ is said to be

   (a) Analytic (b) Harmonic

   (c) Continuous (d) Homomorphic

3. A mapping by the conjugate of an analytic function with a nonvanishing derivative is said to be

(a) Conformal

(b) Holomorphic

(c) Indirectly conformal

(d) Conjugate

4. If $w = S(z) = \frac{az+b}{cz+d}$, then $z = S^{-1}(w)$ is

(a) $\frac{dw-b}{-cw+a}$ (b) $\frac{aw+b}{cw+d}$

(c) $\frac{dw+b}{-cw+a}$ (d) $\frac{-dw-b}{-cw+a}$

5. If $e^{h(\beta)} = 1$, then $h(\beta)$ must be

(a) 0 (b) a multiple of $2\pi i$

(c) a multiple of 2π (d) a multiple of πi

6. If C is the unit circle $|z| = 1$, then $\int_C \frac{e^z}{z} dz$ is

(a) 0 (b) 1

(c) $2\pi i$ (d) $2\pi e i$

7. "A function which is analytic and bounded in the whole plane must reduce to a constant" — This result is known as

(a) Liouville's Theorem

(b) Morera's Theorem

(c) Cauchy's Theorem

(d) The fundamental theorem of algebra

8. If $f(z)$ is defined and continuous on a closed bounded set E and analytic on the interior of E, then the maximum of $|f(z)|$ on E is assumed

(a) on the interior of E

(b) on the boundary of E

(c) on the set E

(d) on the closure of E

9. The residue of $\frac{e^z}{(z-a)^2}$ at $z = a$ is

(a) e^0 (b) e

(c) e^a (d) e^{a^2}

10. An integral of the form $\int_{-\infty}^{+\infty} \frac{P(x)}{Q(x)} dx$ converges if and only if

(a) deg $Q(x)$ is at least two units higher than deg $P(x)$ andif no pole lies on the real axis

(b) deg $P(x)$ is at least two units higher than deg $Q(x)$ and if no pole lies on the real axis

(c) deg $Q(x)$ is at least one unit higher than deg $P(x)$ and if no pole lies on the real axis

(d) deg $P(x)$ = deg $Q(x)$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove rigorously that the functions $f(z)$ and $\overline{f(\overline{z})}$ are simultaneously analytic.

Or

(b) State and prove Lucas's theorem.

12. (a) Given three distinct points z_2, z_3, z_4 in the extended plane, prove that there exists a unique linear transformation which carries them into 1, 0, ∞ in this order.

Or

(b) Find the linear transformation which carries 0, i, $-i$ into 1, -1, 0.

13. (a) State and prove Cauchy's integral formula.

Or

(b) If the piecewise differentiable closed curve γ does not pass through the point a, prove that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

14. (a) Compute $\int_{|z|=1} e^z z^{-n} dz$.

Or

(b) State and prove the fundamental theorem of algebra.

15. (a) State and prove the residue theorem.

Or

(b) Find the residue of $\frac{z+1}{z^2 - 2z}$ at its poles.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that $u = x^2 - y^2$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function.

Or

(b) State and prove Abel's limit theorem.

17. (a) Obtain a necessary and sufficient condition under which a line integral depends only on the end points.

Or

(b) If $T_1 z = \dfrac{z+2}{z+3}, T_2 z = \dfrac{z}{z+1}$, find $T_1 T_2 z$, $T_2 T_1 z$ and $T_1^{-1} T_2 z$.

18. (a) State and prove Cauchy's theorem for a rectangle.

Or

(b) Compute $\displaystyle\int_{|z|=\rho} \frac{|dz|}{|z-a|^2}$ under the condition $|a| \neq \rho$.

19. (a) Prove that analytic function $f(z)$ has derivatives of all orders which are analytic and can be represented by the formula $f^{(n)}(z) = \frac{n!}{2\pi i} \int_C \frac{f(\xi)d\xi}{(\xi - z)^{n+1}}$.

Or

(b) State and prove Taylor's theorem.

20. (a) State and prove the argument principle.

Or

(b) Evaluate $\int_0^\infty \frac{\cos x}{1+x^2} dx$.