(7 pages) **Reg. No. :** ....

Code No. : 40344 E **Sub. Code : JAMA 11/ SAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First/Third Semester

Mathematics — Allied

ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If α, β, γ are the roots of the equation $x^3 + 2x - 6 = 0$, then the value of $\alpha\beta\gamma$ is

(a) 0 (b) 2

(c) 6 (d) –6.

2. If $f(x) = 0$ is a reciprocal equation of first type and odd degree, then ———— is a factor of $f(x)$.

(a) $x+1$ (b) $x-1$

(c) x^2-1 (d) x^2+1.

3. The equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ will be transformed by decreasing the roots by unity into the reciprocal equation

(a) $x^4 + x^3 + x^2 + x + 1 = 0$

(b) $x^4 - x^3 - x^2 - x + 1 = 0$

(c) $x^4 - x^3 + x^2 + x - 1 = 0$

(d) $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$.

4. If all the roots $f(x) = 0$ are real then all the roots $f'(x) = 0$ are

(a) imaginary (b) real and distinct

(c) real (d) real and imaginary.

5. The characteristic polynomial of $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ is

(a) $x^2 - 2x + 5 = 0$ (b) $x^2 + 2x + 5 = 0$

(c) $-x^2 - 2x + 5 = 0$ (d) $x^2 - 2x - 5 = 0$.

6. If the eigen values of a square matrix A are 1, 2, 3, then the eigen values of A^2 are

(a) 1, 4, 9 (b) 2, 4, 6

(c) –1, –4, –9 (d) 1, 1/2, 1/3.

7. The solution of $p^2 - 3p + 2 = 0$ is

(a) $(y - 2x + c_1)(y + x + c_2) = 0$

(b) $(y - 2x - c_1)(y - x - c_2) = 0$

(c) $(y - 3x - c_1)(y + 3x - c_2) = 0$

(d) $(y - 4x - c_1)(y + 4x - c_2) = 0$.

8. The partial differential equation obtained by eliminating 'f' from $z = f(x^2 + y^2)$ is

(a) $py^2 = qx^2$ (b) $px^2 = qy^2$

(c) $py = qx$ (d) $px = qy$.

9. $L[e^{-t}t^3] =$ ____________

(a) $\frac{1!}{(s+1)^4}$ (b) $\frac{2!}{(s+1)^4}$

(c) $\frac{4!}{(s+1)^2}$ (d) $\frac{3!}{(s+1)^4}$.

10. $L^{-1}\left[\frac{1}{(s-4)^5}\right] =$ ____________

(a) $\frac{e^{4t}\, t^5}{25}$ (b) $\frac{e^{4t}\, t^4}{24}$

(c) $\frac{e^{-4t}\, t^4}{24}$ (d) $\frac{e^{-4t}\, t^5}{25}$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in arithmetic progression.

Or

(b) Form the equation one of whose roots is $\sqrt{2} + \sqrt{3}$.

12. (a) Diminish the roots of $x^4 + 3x^3 - 2x^2 - 4x - 3 = 0$ by 3.

Or

(b) Find the positive root of $x^3 - 6x + 4 = 0$ correct to two decimal places by Newton's method.

13. (a) Show that if λ is an eigen value of a non-singular matrix A, then $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

Or

(b) Show that $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 2A - 5I = 0$ and hence find A^{-1}.

14. (a) Solve : $y = 2px + y^2p^3$.

Or

(b) Find the partial differential equation by eliminating the arbitrary function $xyz = \phi\left(x^2 + y^2 - z^2\right)$.

15. (a) Find $L\left[t \sin^2 t\right]$.

Or

(b) Find $L^{-1}\left[\log\left(\frac{s^2+9}{s^2+1}\right)\right]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the equation
$$x^4 - 8x^3 + 7x^2 + 36x - 36 = 0,$$
given that two of its roots are equal in magnitude and opposite in sign.

Or

(b) Solve : $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$.

17. (a) Solve $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing its second term.

Or

(b) Find the positive root of $x^3 - x - 3 = 0$ correct to 2 decimal places by using Horner's method.

18. (a) Find the eigen values and the eigen vector of
$$A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}.$$

Or

(b) Verify Cayley-Hamilton theorem and hence find A^{-1} for the matrix $A = \begin{bmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$.

19. (a) Solve :

(i) $xp^2 - 2py + x = 0$.

(ii) $q - p = y - x$.

Or

(b) Solve : $(x + y) zp + (x - y) zq = x^2 + y^2$.

20. (a) Find :

(i) $L\left[\frac{e^{3t} - e^{-2t}}{t}\right]$.

(ii) $L^{-1}\left[\frac{2(s+1)}{(s^2 + 2s + 2)^2}\right]$.

Or

(b) Using Laplace transform solve $y'' + 6y' + 5y = e^{-2t}$ given that $y(0) = 0$ and $y'(0) = 1$.

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