(6 pages) **Reg. No. :** ....

Code No. : 8758 **Sub. Code : KMAM 13**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2016.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. If the two functions $f(x)$ and $g(x)$ are defined on an interval $[a,b]$ and have the property that one is a constant multiple of the other, then they are said to be ———— on $[a,b]$.

   (a) non linear

   (b) linear

   (c) linearly not, independent

   (d) linearly independent

2. If $R(x)$ is identically zero, then the second order linear differntial equation is reduces to the ———— equation

   (a) discontinuous (b) conitinuous

   (c) homogeneous (d) non homogeneous

3. A point that is not an ordinary point of the second order differential equation is called ————

   (a) Ordinary point (b) Fixed point

   (c) multi point (d) Singular point

4. An infinite series of the form $a_0 + a_1x + a_2x^2 + \cdots$ is called a ———— series of x

   (a) Power (b) taylor

   (c) light (d) infinite

5. A singular point of the second order differential equation is said to be ———— if the function $(x-x_0)P(x)$ and $(x-x_0)^2Q(x)$ are analytic.

   (a) fixed (b) singular

   (c) multi (d) regular

6. The procedure for finding the Frobenius series solution is known as the method of ————

   (a) Taylors (b) Frobenius

   (c) Simmons (d) Infinites

7. we often expressed by saying that $P_0(x), P_1(x), P_2, \ldots, P_n(x), \ldots$ is the sequence of ———— functions on the interval $-1 \le x \le 1$.

(a) orthogonal (b) Bessel

(c) polynomial (d) Mixed

8. The Bessel equation is ————

(a) $x^2y'' + xy' + (x - p^2)y = 0$

(b) $x^2y'' + xy' + (x^2 - p^2)y = 0$

(c) $x^2y' + xy' + (x^2 - p^2)y = 0$

(d) $x^2y + xy' + (x^2 - p^2)y = 0$

9. By solving the linear system using anxillary equation. If m_1 and m_2 are distinct real numbers, then the roots are ————

(a) same (b) distinct

(c) trivial (d) nontrivial

10. The equations $\frac{dx}{dt} = x(a - by)$ and $\frac{dy}{dt} = -y(c - dy)$ are called ———— equations.

(a) Bessel

(b) linear

(c) Volterra's prey-predator

(d) Non linear

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that e^x and e^{-x} are linearly independent solutions of $y'' - y' = 0$ on any interval.

Or

(b) The equation $xy'' + 3y' = 0$ has the obvious solution $y_1 = 1$. Find y_2 and the general solution.

12. (a) Show that polynomial and the function e^x are analytic at all points and give example.

Or

(b) Solve the equation $y' = x - y, y(0) = 0$.

13. (a) Find the singular point for the equation $(3x+1)xy'' - (x+1)y' + 2y = 0$.

Or

(b) Find the singular point for the equation $x^2y'' + (2 - x)y' = 0$.

14. (a) Find the three terms of the Legendre series of $f(x) = e^x$.

Or

(b) Show that the change of variables $t = s^2$ leads to $\Gamma\left(\frac{1}{2}\right) = \int_0^\infty t(-1/2)\, e(-t)\, dt$.

15. (a) Prove that if the solutions of the homogeneous system are linearly independent on $[a, b]$, then the system is the general solution of homogeneous solution of this interval

Or

(b) Show that the condition $a_2 b_1 > 0$ is sufficient, but not necessary, for the system to have two real valued linearly independent solutions of the linear system.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that $y = c_1 e^x + c_2 x e^{2x}$ is the general solution of $y'' - 4y' + 4y = 0$ on any interval.

Or

(b) Find the two linearly independent solution of $x^2 y'' - 2y = 0$ on the interval [1,2], determine the particular solution satisfying the initial conditions $y(1) = 1, y'(1) = 8$.

17. (a) Verify that if $f(x)$ is analytic at x_0 and $f^{-1}(x)$ is continuous inverse, then $f^{-1}(x)$ is analytic at $f(x_0)$ if $f'(x_0) \neq 0$.

Or

(b) Write about series solution of first order equations with examples.

18. (a) Bessel equation of order $p = 1$ is $x^2 y'' + xy' + (x^2 - 1)y = 0$. Show that $m_1 - m_2 = 2$ and that the equation has only one Frobenius series solution. Then find it.

Or

(b) Find two independent Frobenius series solution of each of the following equations.

(i) $xy'' + 2y' + xy = 0$.

(ii) $xy'' - y' + 4x^3 y = 0$.

19. (a) Express $J_2(x), J_3(x), J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

Or

(b) Write the properties of Bessel function.

20. (a) Explain about linear systems.

Or

(b) Show that the Wronskian of the two solutions in distinct complex roots is given by $W(t) = (A_1 B_2 - A_2 B_1)e^{2at,}$ and prove that $A_1 B_2 - A_2 B_1 \neq 0$.

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