(7 pages) **Reg. No. :** ....

Code No. : 40572 E **Sub. Code : SMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Fifth Semester

Mathematics — Core

REAL ANALYSIS - II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In any metric space, the diameter of the empty set Φ is ______________

   (a) 0

   (b) 1

   (c) $-\infty$

   (d) $+\infty$

2. If $A = \left\{0, 1, \frac{1}{2}, \ldots, \frac{1}{n}, \ldots\right\}$, then $\operatorname{Int} A =$ ______________

   (a) 0 (b) 1

   (c) $\frac{1}{n}$ (d) ϕ

3. In R with usual metric, $(a, b]$ is ______________ interval

   (a) an open

   (b) a closed

   (c) both open and closed

   (d) neither open nor closed

4. The set of irrational number in R is ______________

   (a) open (b) closed

   (c) dense (d) complete

5. If $f : (0, 1] \to R$ is a function defined by $f(x) = \frac{1}{x}$, then f in $(0, 1]$ is ______________

   (a) both continuous and uniformly continuous

   (b) continuous but not uniformly continuous

   (c) not continuous

   (d) uniformly continuous

6. For any $n \in Z$, oscillation $w(f, n) =$ ____________

(a) n (b) 0

(c) 1 (d) ∞

7. R means

(a) connected

(b) not connected

(c) compact

(d) neither connected nor compact

8. The set $\{[0, n) | n \in N\}$ is an open cover for ____________

(a) R (b) N

(c) $[0, \infty)$ (d) $(-\infty, \infty)$

9. If f is differentiable at a point c, then f is ____________

(a) continuous at c

(b) not continuous at c

(c) uniformly continuous at c

(d) both continuous and uniformly continuous at c

10. $S(P, f) = \sum_{K=1}^{n} f(t_k) \Delta x_k$ is called as ____________

(a) Riemann-stieltjes sum

(b) Riemann-sum

(c) Riemann integral

(d) Riemann-stieltjes integral

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If (M, d) is a metric space and if $d_1(x, y) = \min\{1, d(x, y)\}$, then prove that d_1 is a metric on M.

Or

(b) Prove that in any metric space, the intersection of a finite number of open sets in open.

12. (a) Prove that in any metric space, every closed ball is a closed set.

Or

(b) A subset A of a complete metric space M is complete iff A is closed.

13. (a) Define a continuous functions. Prove that the composition of two continuous functions is continuous.

Or

(b) If f is monotonic on $[a, b]$, then prove that the set of discontinuities of f is countable.

14. (a) Prove that any continuous image of a connected set is connected.

Or

(b) Prove that a subset A of R is compact iff A is closed and bounded .

15. (a) If f and g are defined on (a, b) and differentiable at c, then prove that f/g is also differentiable at c if $g c \neq 0$ and find $(f/g)'(c)$.

Or

(b) State and prove the mean value theorem for derivatives.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) If (M, d) is a metric space and if $d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$, then prove that d_1 is a metric on M.

(ii) Prove that in any metric space (M, d) each open ball is an open set.

Or

(b) If (M, d) is metric space and if $A, B \leq M$, then prove the followings:

(i) $Int\ A$ = union of all open sets contained in A

(ii) $A \subseteq B \Rightarrow Int\ A \subseteq Int\ B$

(iii) $Int(A \cap B) = Int\ A \cap Int\ B$

(iv) $Int\ (A \cup B) \supseteq Int\ A \cup Int\ B$.

17. (a) Show that l_2 is complete.

Or

(b) State and prove Baire's category theorem.

18. (a) Prove that f is continous iff the inverse image of every open set is open.

Or

(b) Show that $f: R \to R$ is continuous at $a \in R$ iff $w(f, a) = 0$.

19. (a) Prove that a subspace of R is connected iff it is an interval.

Or

(b) State and prove Heine-Borel theorem.

20. (a) State and prove the chain rule for derivatives.

Or

(b) State and prove Taylor's theorem.

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