(8 Pages) **Reg. No. : ....**

Code No. : 6317 **Sub. Code : PMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2021

Third Semester

MATHEMATICS — CORE

TOPOLOGY — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answers :

1. Let Y be a sub space of X. If U is open in Y then U is ———— in X.

   (a) Open (b) Closed

   (c) Clopen (d) None

2. A subset of a topological space is closed if and only if it contains ————

(a) All its limit points

(b) Only one of its limit points

(c) None of its limit points

(d) None

3. Let A be a subset of a topological space X and $x \varepsilon X$. If every neighbourhood of X intersects A then x is ————

(a) An interior point of A

(b) A limit point of A

(c) A closed point of A

(d) None

4. Let X and Y be topological spaces; let $p : x \to y$ be a surjective map. The map p is called a ———— map, provided a subset U of Y is open in Y if and only if $p^{-1}(U)$ is open in X.

(a) open

(b) inverse

(c) closed

(d) quotient

5. The set of limit points of the set $B = \{1/n \mid n \varepsilon z\}$ is

(a) $\{\,\}$

(b) $\{0\}$

(c) $\{1\}$

(d) $\{2\}$

6. ________ is a compact space.

(a) R (b) Q

(c) (2, 3) (d) [4, 5]

7. A subspace of Hausdorff space

(a) is Hausdorff

(b) is not Hausdorff

(c) need not be Hausdorff

(d) none

8. A finite cartesion product of connected space

(a) Is always connected

(b) Need not be connected

(c) Is open

(d) None

9. Which is the following is true

(a) Every regular space is Hausdorff

(b) Every regular space is normal

(c) Every Hausdorff space is regular

(d) Every Hausdorff space is normal

10. Which of the following is true?

(a) A regular space is completely regular

(b) Every topological space has a metrization

(c) Every subspace of a completely regular space is regular

(d) None

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b)

Each answer should not exceed 250 words.

11. (a) Define product topology of two topological spaces and give examples.

Or

(b) Prove that every finite point set is a Hausdorff space is closed.

12. (a) State and prove the pasting lemma.

Or

(b) Let A be a subset of a topological space X and let A' be the set of all limit points of A. Prove that $A = A \cup A'$.

13. (a) State and prove sequence lemma.

Or

(b) Prove that the topologies on R^n induced by the Euclidean metric d and the square metric ρ are the same as the product topology on R^n.

14. (a) Let $f : X \to Y$ be a bijective continuous function. If X is compact and Y is Hausdorff prove that f is a homeomeorphism.

Or

(b) Prove that the product of finitely many compact spaces is compact.

15. (a) Prove that compactness implies limit point compact but not conversely.

Or

(b) Prove that R^n is locally compact but R^w is not locally.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)
Each answer should not exceed 600 words.

16. (a) Define finite compliment topology and a convergent sequence in a topological space. What are the closed sets in it?

Or

(b) If $\{J_\alpha\}$ be a family of topologies on X, show that $\bigcap J_\alpha$ is a topology on X. Is $\bigcup J_\alpha$ a topology on X?

17. (a) Let X and Y be topological spaces and let $f: X \to Y$ be a mapping. Prove that the following are equivalent.

(i) f is continuous

(ii) for every subset A of X, $f(A) \subset \overline{f(A)}$

(iii) for every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

Or

(b) If X_α is a Hausdorff space for each α, prove that πX_α is a Hausdorff space in both the box and product topology.

18. (a) Let $f : X \to Y$. If the function f is continuous prove that for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ and that the converse holds if X is metrizable.

Or

(b) Prove that R^W is connected in the product topology that not in the box topology.

19. (a) State and prove tube lemma.

Or

(b) Let A be a connected subset of X. If $A \subset B \subset \overline{A}$, then prove that B is also connected.

20. (a) Let X be a metrizable space. Prove that the following are equivalent.

(i) X is compact

(ii) X is limit point compact

(iii) X is sequentially compact.

Or

(b) Let X be a space. Prove that X is locally compact Hausdorff if and only if there exists a space Y satisfying the following conditions.

(i) X is a subspace of Y

(ii) The set $Y - X$ consists of a single point

(iii) Y is a compact Hausdorff space. If Y and Y' are two spaces satisfying these conditions then there is a homoeomorphism of Y with Y' that equals the identity map on X.
