

- (b) If K is a field and if $\sigma_1, \sigma_2, \dots, \sigma_n$ are distinct automorphisms of K , then prove that it is impossible to find elements $a_1 \dots a_n$ not all 0, in K , such that
- $$a_1 \sigma_1(u) + \dots + a_n \sigma_n(u) = 0 \text{ for all } u \in K.$$
18. (a) Suppose that the field F has all the n th roots of unity and suppose that $a \neq 0$ is in F . Let $x^n - a \in F(x)$ and let K be the splitting field over F . Then prove that
- $K = F(U)$ where U is any root of $x^n - a$
 - the Galois group of $x^n - a$ over F is abelian.

Or

- (b) Show that S_n is not solvable for $n \geq 5$.
19. (a) If F is a finite field and $a \neq 0, b \neq 0$ are two elements of F . Show that we can find elements $a, b \in F$ such $1 + \alpha a^4 + \beta b^2 = 0$.

Or

- (b) Prove that every positive integer can be expressed as a sum of squares of four integers.
20. (a) State and prove Stone's theorem.
- Or
- (b) State and prove the Jordan Holder theorem.

Reg. No. :

Code No. : 8057

Sub. Code : C 31 M

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2011.

Third Semester

Mathematics

FIELD THEORY AND LATTICES

(For those who joined in July 2008 and afterwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

- Define an extension field and degree of the extension.
- Describe $F(a)$.
- Define a fixed field.
- The fixed field of G is a sub field of K . Prove.
- Define Galois group.
- Express the polynomial $x_1^3 + x_2^3 + x_3^3$ in the elementary symmetric functions in x_1, x_2, x_3 .
- Define a finite field with an example.
- Under what condition S_p is a field?

9. Find all the elements a in Q_0 . Such that a^{-1} is also Q_0 .
10. For all $x, y \in Q$. Show that $N(xy) = N(x)N(y)$.

PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) If a, b in k are algebraic over F then prove that $a + b$ and a/b (if $b \neq 0$) are all algebraic over F .
- Or
- (b) State and prove remainder theorem.
12. (a) If K is a finite extension of F then prove that $G(K, F)$ is a finite group and show that its order satisfies $O(G(K, F))$.

Or

- (b) If K is a sub field of Ω and S set of automorphism of K , prove that the fixed field of S that of S , the sub group of all automorphisms of K generated by S are identical.
13. (a) Prove that the commutator sub group a' is normal.
- Or
- (b) Prove that the general polynomial of degree $n > 5$ is not solvable by radicals.
14. (a) Let F be a finite field with Z elements and suppose that $F \subset K$ where K is also a finite field. Prove that K has q^n elements where $n = [K : F]$.

Or

- (b) Prove that there are exactly 24 elements a in A such that a^{-1} is also in H . Determine all of them.

15. (a) State and prove criterion for isomorphism for lattices.

Or

- (b) Prove that any ring for which there exists a point p such that $pa = a^p a$ for every a in the ring is commutative.

PART C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) If L is an algebraic extension of K and K is an algebraic extension of F . Prove that L is an algebraic extension of F .

Or

- (b) Show that any two fields of the same polynomial over a given field F are isomorphic by an isomorphism leaving every element of field.

17. (a) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F .

Or