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Reg. No. :

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M.Sc.(CBCS) DEGREE EXAMINATION, APRIL 2011.

Second Semester

Mathematics

ALGEBRA – II

(For those who joined in July 2008 and afterwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

1. Define a linear functional.
2. Define a sub module.
3. Show that the characteristics roots of

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \text{ are the fourth roots of unity.}$$

4. Define a invertible matrix.

5. Let $T:V \rightarrow V$ is linear. Show that Kernal of T is invariant under T .
6. What is the minimal polynomial of nilpotent matrix A of index k .
7. Determine a Jordan Canonical form for a matrix of order 5 whose minimal polynomial is $p(x)=(x-2)^2$.
8. Let B be a 4×4 matrix with minimal polynomial $p(x)=(x^2-3)(x^2+1)$. Find the rational canonical form for B if B is a matrix over the complex field.
9. Define a symmetric matrix $A \in F_n$.
10. Write down the matrix of the quadratic form :
 $x_1^2 - 2x_2^2 + 5x_3^2 - 2x_1x_2 + 4x_2x_3 - 16x_3x_1$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if T is a homomorphism of M into N and $K(T) = \{x \in M / xT=0\}$, then $K(T)$ is a submodule of M .

Or

- (b) Prove that any unital irreducible R -module is cyclic.

12. (a) If $\lambda \in F$ is a characteristics root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristics root of $q(T)$.

Or

- (b) If $T, S \in A(V)$ and if S is regular, then prove that T and STS^{-1} have the same minimal polynomial.
13. (a) Find the set D of all matrices commuting with $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

Or

- (b) Prove that

$$M = \left\{ \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \right\} \text{ is an irreducible set.}$$

14. (a) Find all possible Jordan canonical forms for a matrix of order 5 whose minimal polynomial is $p(x) = (x-2)^2$.

Or

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- (b) Let $T: V \rightarrow V$ has characteristics polynomial $g(x) = (x+8)^4(x-1)^3$ and minimal polynomial $p(x) = (x+8)^3(x-1)^2$. Find the Jordan canonical form of the matrix representation of T .

15. (a) If N is normal and if for $\lambda \in F, v(N-\lambda)^k = 0$ then prove that $vN = \lambda v$.

Or

- (b) If N is normal and $AN = NA$ then prove that $AN^* = N^*A$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If the system of homogeneous linear equations

$$a_{11}x_1 + \dots + a_{1n}x_n = 0$$

$$a_{21}x_1 + \dots + a_{2n}x_n = 0$$

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$$a_{m1}x_1 + \dots + a_{mn}x_n = 0$$

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[P.T.O.]

Where $G_y \in F$ of rank r , then prove that there are $n-r$ linearly independent solutions in $F^{(n)}$.

Or

- (b) If F is the field of real numbers, find $A(W)$ where W is spanned by $(1,2,3)$ and $(0,4,-1)$.
17. (a) If A is an algebra with unit element over F , then prove that A is isomorphic to the subalgebra of $A(V)$ for some vector space V over F .
- Or
- (b) Prove that if V is finite dimensional over F , then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.
18. (a) If M is a irreducible set of linear transformations on V and if $D = \{T \in A(V) \mid Tm = mT \text{ for all } m \in M\}$ then prove that D is a division ring.

Or

- (b) If M is a commutative set of elements of $A(V)$ such that $m \in M$ has all its characteristic roots in F then prove that there is a $C \in A(V)$ such that every CmC^{-1} is in triangular form.

19. (a) Prove that the elements S and T in $A_F(V)$ are similar if and only if they have the same elementary divisors.

Or

- (b) Prove that if T in $A_F(V)$ has a minimal polynomial $p(x) = q(x)^e$ where $q(x)$ is a monic, irreducible in $F[x]$, then a basis of V can be found in which the matrix of T is of the form

$$J_i = \begin{bmatrix} c(q(x)^{e_1}) & & & \\ & c(q(x)^{e_2}) & & \\ & & \ddots & \\ & & & c(q(x)^{e_r}) \end{bmatrix}$$

where $e = e_1 + e_2 + \dots + e_r$.

20. (a) Prove that if $\{v_1, v_2, \dots, v_n\}$ is an orthonormal basis of V and if the matrix of $T \in A(V)$ in this basis is (α_{ij}) then the matrix of T^* in this basis is (β_{ij}) where $\beta_{ij} = \overline{\alpha_{ji}}$.

Or

- (b) Let N be a normal transformation and suppose that λ and μ are two distinct characteristic roots of N . If v, w are in V and are such that $vN = \lambda v, wN = \mu w$, then prove that $(v, w) = 0$.