(7 pages) **Reg. No. :** ....

Code No. : 7134 **Sub. Code : PMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $u = 2x - x^3 + 3xy^2$ then Δu is

(a) $12x$ (b) 0

(c) $6x - 6$ (d) $6xy + 6x$.

2. If $P(z) = \alpha_3 (z - \alpha_1)(z - \alpha_2)(z - \alpha_3)$ then $\dfrac{P'(z)}{P(z)}$ is

(a) $\dfrac{1}{z - \alpha_1}$ (b) $\dfrac{1}{z - \alpha_2}$

(c) $\dfrac{1}{z - \alpha_3}$ (d) α_3.

3. The complex form of the Cauchy–Riemann equation is

(a) $\dfrac{\partial f}{\partial x} = i\dfrac{\partial f}{\partial y}$ (b) $\dfrac{\partial f}{\partial x} - i\dfrac{\partial f}{\partial y} = 0$

(c) $\dfrac{\partial f}{\partial x} = -i\dfrac{\partial f}{\partial y}$ (d) $\dfrac{\partial f}{\partial y} = -i\dfrac{\partial f}{\partial x}$.

4. The points z and z^{α} are symmetric w.r.t. the circle C through z_1, z_2, z_3 if and only it

(a) $(z^*, z_1, z_2, z_3) = (z, z_1, z_2, z_3)$

(b) $(z^*, z_1, z_2, z_3) = -(z, z_1, z_2, z_3)$

(c) $(z, z_1, z_2, z_3) = z + z^*$

(d) $\overline{(z, z_1, z_2, z_3)} = (z^*, z_1, z_2, z_3)$.

5. $\displaystyle\int_{|z-2|=5} \frac{dz}{z-3}$ is

(a) 1 (b) 0

(c) $2\pi i$ (d) $-2\pi i$.

6. The index of the point a w.r.t. the cure γ is

(a) $\displaystyle\int_{\gamma} \frac{dz}{z-a}$ (b) $\displaystyle 2\pi i \int_{\gamma} \frac{dz}{z-a}$

(c) $\displaystyle\frac{1}{2\pi i}\int_{\gamma} \frac{dz}{z-a}$ (d) $\displaystyle\int_{\gamma} (z-a)\, dz$.

7. "A function which is analytic and bounded in the whole plane must reduce to a constant" — This result is known as

(a) Morera's theorem
(b) Liouville's theorem
(c) Fundamental theorem of algebra
(d) Cauchy's theorem.

8. $\int\limits_{|z|=1} e^z \cdot z^{-n}\, dz$ is

(a) 0 (b) $2\pi i$
(c) $\frac{2\pi i}{(n-1)!}$ (d) $\frac{2\pi i}{n!}$.

9. The residue of $\frac{e^z}{(z-a)^2}$ at $z = a$ is

(a) e^a (b) e^{2a}
(c) e^{-2a} (d) e^{-a}.

10. If f has a pole of order h then

(a) f_1/f has the residue $-h$
(b) f_1/f has the residue h
(c) f/f_1 has the residue h
(d) f_1/f has the residue $\frac{-h}{h}$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) If all zeros of a polynomial $P(z)$ lie in a half plane, prove that all zeros of the derivative $P'(z)$ lie in the same half plane.

Or

(b) Verify Cauchy-Riemann's equations for the function z^3.

12. (a) Show that any linear transformation which transforms the real axis into itself can be written with real coefficients.

Or

(b) Find the linear transformation which carries $0, i, -i$ into $1, -1, 0$.

13. (a) Obtain Cauchy's integral formula.

Or

(b) Compute $\int\limits_{|z|=2} \frac{dz}{z^2+1}$.

14. (a) State and prove Liouville's theorem.

Or

(b) State and prove the fundamental theorem of algebra.

15. (a) State and prove the residue theorem.

Or

(b) State and prove the Rouche's theorem.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Define an analytic function with an example. Prove that the functions $f(z)$ and $\overline{f(\bar{z})}$ are simultaneously analytic.

Or

(b) (i) Show that $\sum_{0}^{\infty} a_n z^n$ and $\sum_{1}^{\alpha} n a_n z^{n-1}$ have the same radius of convergence.

(ii) If $\sum_{0}^{\alpha} a_n$ converges, prove that $f(z) = \sum_{n}^{\alpha} a_n z^n \to f(1)$ as $z \to 1$ in such a way that $|1-z|/(1-|z|)$ remains bounded.

17. (a) Prove that the line integral $\int_{\gamma} p\,dx + q\,dy$ defined in Ω depends only on the end points of γ if and only if there exists a function $U(x, y)$ in Ω with the partial derivatives $\frac{\partial U}{\partial x} = p$, $\frac{\partial U}{\partial y} = q$.

Or

(b) (i) Define the cross ratio (z_1, z_2, z_3, z_4). Show that the cross ratio is invariant under linear transformation.

(ii) Prove that a linear transformation carries circles into circles.

18. (a) State and prove Cauchy's theorem for a rectangle.

Or

(b) Show that the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$ and hence show that the winding number $n(\gamma, a)$ is an integer. Also show that $n(-\gamma, a) = -n(\gamma, a)$.

19. (a) State and prove Taylor's theorem.

Or

(b) State and prove Weierstrass theorem for essential singularity of an analytic function.

20. (a) State and prove the argument principle.

Or

(b) Evaluate $\int_{-a}^{a} \frac{x^2 - x + 2}{x^4 + 10x^2 + q} \, dx$.