(6 pages) **Reg. No. :** ......................................

**Code No. : 8365** **Sub. Code : HMAM 33**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2016.

Third Semester

Mathematics

DIFFERENTIAL GEOMETRY

(For those who joined in July 2012 – 2015)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. As P moves along a curve, the arc-rate at which the osculating plane turns about the tangent is called the —————.

   (a) torsion (b) curvature

   (c) normal line (d) none

2. The normal in a direction orthogonal to the osculating plane in the ————— line.

   (a) tangent (b) binormal

   (c) normal (d) none

3. The osculating circle at a point P on a curve has ————— point of contact with the curve at P.

   (a) one (b) two

   (c) three (d) no

4. The position vector of center of spherical curvature is $\overline{C}$ = —————.

   (a) $\overline{r} + \rho\overline{n} + \sigma\rho'\overline{b}$ (b) $\overline{r} + \rho\overline{b}$

   (c) $\overline{r} + \rho\overline{t}$ (d) $\overline{r} + \rho\overline{n}$

5. Since $\overline{r}_1 \times \overline{r}_2 \neq 0$, parametric curves of different systems ————— each other.

   (a) touch (b) cannot touch

   (c) intersect (d) none

6. If $w$ is the angle between the parametric curves, then $\sin w$ = —————.

   (a) $EG - F^2$ (b) $F/\sqrt{EG}$

   (c) $G/\sqrt{EG}$ (d) none

7. Geodesics are ————— of any particular parametric representation of the surface.

   (a) dependent (b) independent

   (c) the curves (d) none

8. At every point on a geodesic, its principal normal is ———————— to the surface.

(a) orthogonal (b) parallel

(c) normal (d) none

9. If $k_a$ and $k_b$ are two principal curvatures at a point on a surface, then the Gaussian curvature $k =$ ————————.

(a) $\sqrt{k_a k_b}$ (b) $k_a k_b$

(c) $k_a + k_b$ (d) none

10. A point on a surface at which $\frac{L}{E} = \frac{M}{F} = \frac{N}{G}$ is called ———————— point.

(a) umbilic (b) parabolic

(c) elliptic (d) hyperbolic

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find the arc length of the curve $\bar{r} = (a\cos^3 t, a\sin^3 t, 0)$.

Or

(b) Show that the length of the common perpendicular $d$ of the tangents at two near point distance $s$ apart is approximately given by $d = k\tau s^3/12$.



12. (a) Derive the locus of center of spherical curvature.

Or

(b) Show that the ratio of the curvature to the torsion is constant at all points on a Helix.

13. (a) Find the angle between the parametric curves.

Or

(b) Find the coefficients of the direction which makes an angle $\pi/2$ with the direction whose coefficients are $(l, m)$.

14. (a) Prove that the curves of the family $v^3/u^2 = c$, a constant are geodesics on a surface with metric $v^2du^2 - 2uv\,du\,dv + 2u^2dv^2$.

Or

(b) Prove that every helix on a cylinder is a geodesic.

15. (a) Prove that if the orthogonal trajectories of the curves $v = c$ are geodesics, then $H^2/E$ is independent of $u$.

Or

(b) State and prove Meusiner's theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Derive Serret-Frenet formulae.

Or

(b) Show that the necessary and sufficient condition that a curve lies on a sphere is $\frac{\rho}{\sigma} + \frac{d}{ds}(\sigma\rho') = 0$.

17. (a) Show that the intrinsic equation of $x = a(\theta - \sin\theta)$, $= a(1 - \cos\theta)$, $z = 0$ are $l^2 s^2 = 16a^2$, $\tau = 0$.

Or

(b) State and prove fundamental existence theorem for space curves.

18. (a) Calculate the fundamental magnitudes for the right helicoid given by $x = u\cos v$, $y = u\sin v$, $z = cv$.

Or

(b) Show that a proper pametric transformation either leaves every normal unchanged or reverses every normal.



19. (a) (i) Discuss about orthogonal trajectories.

(ii) On the paraboloids $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the plane $z$ = constant.

Or

(b) Derive the differential equations satisfied by the geodesics on a given surface.

20. (a) Derive the Liouville's formula for Kg.

Or

(b) (i) Find the geodesic curvature of the parametric curve $v$ = constant.

(ii) State and prove Rodrigue's formula.

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