(8 pages) **Reg. No. :** .....

Code No. : 8370 **Sub. Code : HMAE 41**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2014.

Fourth Semester

Mathematics

Elective — GRAPH THEORY

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following is true?
   (a) An empty graph may contain more than one vertex
   (b) Every walk is a path
   (c) Subgraph of a connected graph is connected
   (d) There is a 3-regular graph on 7 vertices

2. Which of the following is true?
   (a) Every cycle is a trail
   (b) A bipartite graph always has an even cycle
   (c) If a graph is acyclic, then it is a tree
   (d) Number of spanning trees of a tree on v vertices is $v-1$

3. Which of the following is true?
   (a) A connected graph may have more than only one block
   (b) A tree has only one block
   (c) In a connected graph any two vertices lie on a common cycle
   (d) If G is a block, then any two edges lie on a common cycle

4. Which of the following is true?
   (a) Every Euler graph is Hamiltonian
   (b) K_6 is both Eulerian and Hamiltonian
   (c) Every Hamiltonian graph is Eulerian
   (d) No tree is Hamiltonian

5. Which of the following is true?

(a) Every graph has a perfect matching

(b) $K_{4,4}$ has a perfect matching

(c) If G is bipartite, then $\chi'(G)=\Delta-1$

(d) K_{13} has a perfect matching

6. Which of the following is true?

(a) $K_{n,n}$ has a perfect matching

(b) A 3-regular graph need not have a perfect matching

(c) A cycle on 9 vertices has a perfect matching

(d) Every graph on $2n$ vertices has a perfect matching

7. Which of the following is true?

(a) K_n has an independent set

(b) Covering number is always less than or equal to the independence number

(c) Every independent set is contained in a maximum independent set

(d) The Ramsey number $r(2, n)=2$

8. Which of the following is true?

(a) $K_{3,3}$ has no independent set

(b) Independence number is always less than or equal to the covering number

(c) For a graph on v vertices the independence number may be v

(d) The Ramsey number $r(3, 3)=5$

9. Which of the following is true?

(a) If G is $k-$critical, then $\delta \geq k-1$

(b) If G is $k-$critical, then $\delta > k$

(c) Chromatic number of a cycle on n vertices is n

(d) None of the above

10. Which of the following is true?

(a) Every 4-chromatic graph has $4n$ vertices.

(b) K_4 has a subdivision of K_5 as a subgraph.

(c) K_5 can be obtained as a subdivision of K_4

(d) None of the above

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each question carries 5 marks.

11. (a) Show that for any graph $\delta \leq \frac{2\varepsilon}{\upsilon} \leq \Delta$.

Or

(b) Prove that a k-regular graph of girth 4 has at least k vertices.

12. (a) Show that a graph is 2-edge connected if and only if any two vertices are connected by at least two edge disjoint paths.

Or

(b) If G is Eulerian, then prove that any trail of constructed by Fleury's algorithm is an Euler tour of G.

13. (a) Show that a tree, has at most one perfect matching.

Or

(b) Let G be a connected graph that is not an odd cycle. Then prove that G has a two edge coloring in which both colors are represented at each vertex of degree at least two.

14. (a) Show that G is bipartite if and only if $\alpha(H) \geq \frac{1}{2}\upsilon(H)$ for every subgraph H of G.

Or

(b) Show that, for all k and l, $r(k,l)=r(l, k)$.

15. (a) In a critical graph prove that no vertex cut is a clique.

Or

(b) For any graph G, prove that $\pi_k(G)$ is a polynomial in k of degree υ with integer coefficients, leading term k^{υ} and constant term 0. Also prove that the coefficients are alternate in sign.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

Each question carries 8 marks.

16. (a) A sequence $d:(d_1, d_2,..., d_n)$ is graphic and $d_1 \geq d_2 \geq ... \geq d_n$, then show that $\sum_{i=1}^{n} d_i$ is even and

$$\sum_{i=1}^{k} d_i \le k(k-1) + \sum_{i=k+1}^{n} \min\{k, d_i\}$$

for $1 \le k \le n$.

Or

(b) Show that if G is a tree with $\Delta \ge k$, then G has atleast k vertices.

17. (a) Prove that $k \le k' \le \delta$.

Or

(b) Define closure of a graph and prove that it is well defined also find the closure of $K_{3,3}$.

18. (a) Let G be a bipartite graph with bipartition (X, Y). Then prove that G contains a matching that saturates ever vertex in X if and only if $N(S) \ge |S|$ for all $S \subseteq X$.

Or

(b) State and prove Vizing's theorem.

19. (a) For any two integers $k \ge 2$ and $l \ge 2$, prove that $r(k,l) \le r(k, l-1) + r(k-1, l)$.

Or

(b) Prove that $r(k,k) \le 2^{\frac{k}{2}}$.

20. (a) Let G be a k-critical graph with a 2-vertex cut $\{u, v\}$, then prove that

(i) $G = G_1 \cup G_2$ where G_i is a $\{u, v\}$ component of type i for $i = 1, 2$

(ii) Both $G_1 + uv$ and $G_2 \cdot uv$ are k-critical.

Or

(b) Prove that for any positive integer k, there exists a k-chromatic graph containing no triangle.
