(8 pages) **Reg. No. :**.....

Code No. : 20581 E **Sub. Code : SMMA 64**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Sixth Semester

Mathematics — Core

DYNAMICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Greatest height attained by a projectile is

(a) $\frac{u^2 \sin^2 \alpha}{2g}$ (b) $\frac{u^2 \cos^2 \alpha}{2g}$

(c) $\frac{u \sin^2 \alpha}{2g}$ (d) $\frac{2u \sin \alpha}{g}$

2. A particle is projected with velocity $80\sqrt{2}$ ft/sec at an elevation of 45° then the time ofp flight is

(a) 2 sec (b) 5 sec

(c) 4 sec (d) 3 sec

3. Momentum is a _____.

(a) constant (b) scalar

(c) vector (d) none of the above

4. When a perfectly elastic sphere impinges on a fixed smooth plane, the angle of reflection = _____.

(a) 90° (b) 45°

(c) 0° (d) angle of incidence

5. The maximum velocity of a particle executing SHM is 1 m/sec and its period is $\frac{1}{5}$ of the second. The amplitude is

(a) $\frac{1}{10}$ m (b) $10\pi m$

(c) $\frac{\pi}{10}m$ (d) $\frac{1}{10\pi}m$

6. If $x = a\cos wt + b\sin wt$, then the constant μ of the SHM is

(a) w (b) $-w$

(c) w^2 (d) $-w^2$

7. The magnitude of the radial component of velocity is

(a) $\dot{r}$ (b) $r\dot{\theta}$

(c) $\ddot{r}$ (d) $r^2\dot{\theta}$

8. For a particle describing a circle of radius a, the acceleration at any point P has the component _____ along the tangent at P.

(a) $a\dot{\theta}^2$ (b) $a\ddot{\theta}$

(c) $a\dot{\theta}$ (d) $a^2\dot{\theta}$

9. (p, r) equation to the spiral is

(a) $p = ar^2$ (b) $p = r\cos\alpha$

(c) $p = r\sin\alpha$ (d) $p = r\tan\alpha$

10. If a particle moves in a central orbit then $r^2\dot{\theta}$ = _____.

(a) h (b) $\frac{h}{2}$

(c) $2h$ (d) $-h$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If the greatest height attained by the particle is a quarter of its range on the horizontal plane through the point of projection then find the angle of projection.

Or

(b) Determine the maximum range on an inclined plane, given the magnitude of the velocity of projection of a particle.

12. (a) A ball of mass 8 kg moving with a velocity of 10 m/sec impinges directly on another ball of mass 24 kg moving at 2 m/sec, in the same direction. If $e = \frac{1}{2}$ then find the velocities after impact. Also calculate the loss in kinetic energy.

Or

(b) A smooth sphere of mass 'm' impinges obliquely on a smooth sphere of mass 'M' which is at rest. Show that if $m = eM$, the directions of motion after impact are at right angles. (e is the coefficient of restitution)

13. (a) A particle moves along a circle with uniform speed. Show that the motion of its projection on a fixed diameter is simple harmonic.

Or

(b) A body moving with SHM has an amplitude 'a' and period 'T'. Show that the velocity 'v' at a distance 'x' from the mean position is given by $v^2T^2 = 4\pi^2\left(a^2 - x^2\right)$.

14. (a) Derive the radial and transverse components of acceleration of a particle.

Or

(b) If a point moves so that its radial velocity is k times its transverse velocity then show that its path is an equiangular spiral.

15. (a) Prove that, in a central orbit, the areal velocity is $\frac{1}{2}pv$.

Or

(b) Find the law of force towards the pole under which the particle describes the curve $r^2 = a^2\cos 2\theta$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that the greatest height which a particle with initial velocity 'v' can reach on a vertical wall at a distance 'a' from the point of projection is $\frac{v^2}{2g} - \frac{ga^2}{2v^2}$.

Or

(b) If v_1 and v_2 be the velocities of a projectile at the ends of a focal chord of its path and u is the velocity at the vertex, prove that $v_1^{-2} + v_2^{-2} = u^{-2}$.

17. (a) Two equal balls are in contact on a smooth table and a third equal ball moving along their common tangent strikes them simultaneously. Prove that $\frac{3}{5}\left(1 - e^2\right)$ of its kinetic energy is lost by impact, e being the coefficient of restitution for each pair of balls.

Or

(b) An elastic ball of mass 'm' falls from a height 'h' on a fixed plane and rebounds. Show that the loss of kinetic energy of impact is $mgh\left(1-e^2\right)$. Show also that the time taken before the particle has finished rebounding is $\sqrt{\frac{2h}{g}}\cdot\left(\frac{1+e}{1-e}\right)$.

18. (a) A particle is moving with SHM has distances x_1, x_2, x_3 in '3' successive intervals of time from its center of oscillation. Show that its period is $\frac{2\pi}{\cos^{-1}\left(\frac{x_1+x_3}{2x_2}\right)}$.

Or

(b) Find the composition of two SHMs of the same period in two perpendicular directions.

19. (a) A particle moves with a uniform speed 'v' along the curve $r=a\left(1+\cos\theta\right)$. Show that its angular velocity about the pole is $\frac{v\sec\frac{\theta}{2}}{2a}$ and the radial component of its acceleration is the constant $\frac{-3v^2}{4a}$.

Or

(b) The velocities of a particle along and perpendicular to the radius from a fixed origin are $\lambda\gamma$ and $\mu\theta$, where λ, μ are constants. Find the path and the accelerations along and perpendicular to the radius vector.

20. (a) A particle moves in an ellipse under a force which is always directed towards its focus. Find the law of force, the velocity at any point of the path and its periodic time.

Or

(b) If p is the perpendicular from the pole on the tangent then prove that $\frac{1}{p^2} = u^2 + \left(\frac{du}{d\theta}\right)^2$.

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