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Reg. No. :

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M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2011.

Second Semester

Mathematics

ANALYSIS — II

(For those who joined in July 2008 and afterwards)

Time : Three hours

Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL the questions.

1. Define Riemann-Stieltjes integral.
2. Define common refinement.
3. Write the fundamental theorem of calculus.
4. Formula for the length of the curve is ———.
5. Define algebra.
6. Define uniformly closed subset of $C(X)$.

7. $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = \underline{\hspace{2cm}}.$

8. $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = \underline{\hspace{2cm}}.$

9. Define trigonometric polynomial.

10. Write the beta function $B(x, y)$.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$, then prove that $f \in \mathbf{R}(\alpha)$.

Or

- (b) If f is continuous on $[a, b]$, then prove that $f \in \mathbf{R}(\alpha)$ on $[a, b]$.

12. (a) Let $f \in \mathbf{R}$ on $[a, b]$. For $a \leq x \leq b$, let

$$F(x) = \int_a^x f(t) dt. \text{ Then prove that } F \text{ is continuous on } [a, b].$$

Or

- (b) Let p and q be positive real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$. Prove the following statement. If

$$f \in \mathbf{R}(\alpha), f \geq 0, g \geq 0 \text{ and } \int_a^b f^p d\alpha = 1 = \int_a^b g^q d\alpha,$$

$$\text{then } \int_a^b fg d\alpha \leq 1.$$

13. (a) Let X be a metric space, $C(X)$ denote the set of all complex valued continuous, bounded functions defined on X . For each $f \in C(X)$ let $\|f\| = \sup_{x \in X} |f(x)|$ be the supremum norm of f . Let $\|f - g\|$ be the metric on $C(X)$. Show that this metric makes $C(X)$ into complete metric space.

Or

- (b) Suppose A is an algebra of functions on a set E , A separates points on E , and A vanishes at no point of E . If x_1, x_2 are distinct points of E , and c_1, c_2 are constants (real if A is a real algebra), then prove that A contains a function f such that $f(x_1) = c_1$ $f(x_2) = c_2$.

14. (a) Let e^x be defined on R^1 by $E(x) = e^x$ and

$$E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!} \text{ then prove that } e^x \rightarrow +\infty \text{ as}$$

$$x \rightarrow +\infty, e^x \rightarrow 0 \text{ as } x \rightarrow -\infty.$$

Or

- (b) If the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$, and

$$f(x) = \sum_{n=0}^{\infty} c_n x^n \text{ then prove that } \sum_{n=0}^{\infty} c_n x^n$$

converges uniformly on $[-R + \varepsilon, R - \varepsilon]$.

15. (a) If $x > 0$ and $y > 0$, then prove that

$$\int_0^1 t^{x-1} (1-t)^{y-1} dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}.$$

Or

- (b) Prove that $\log \Gamma$ is convex on $(0, \infty)$.

PART C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.

Or

- (b) If $f \geq 0$, f is continuous on $[a, b]$ and $\int_a^b f(x) dx = 0$, prove that $f(x) = 0$, for all $x \in [a, b]$.

17. (a) If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and also prove that $\wedge(\gamma) = \int_a^b |\gamma'(t)| dt$.

Or

- (b) Let $\gamma_1, \gamma_2, \gamma_3$, be curves in the complex plane defined on $[0, 2\pi]$ by $\gamma_1(t) = e^{it}$, $\gamma_2(t) = e^{2it}$, $\gamma_3(t) = e^{2\pi i t \sin(1/t)}$. Show that these three curves have the same range.

18. (a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

Or

- (b) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

19. (a) Given in double sequence $\{a_{ij}\}$ $i = 1, 2, 3, \dots$, $j = 1, 2, 3, \dots$, suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_j$ ($i = 1, 2, 3, \dots$) and $\sum b_j$ converges, then prove that $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij}$.

Or

- (b) If $a_0, \dots, a_n$ are complex numbers, $n \geq 1$, $a_n \neq 0$. $P(z) = \sum_0^n a_k z^k$, then prove that $P(z) = 0$ for some complex number z .

20. (a) State and prove Stirling's formula.

Or

- (b) If $f(x) = (\pi - |x|)^2$ on $[-\pi, \pi]$, find the Fourier cosine expression for $f(x)$ and deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.