PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 600 words.

16. (a) State and prove the Binomial theorem.

Or

(b) If α_n is the n^{th} Lucas number, then prove that $\alpha_n < \left(\frac{7}{4}\right)^n$.

17. (a) State and prove the division algorithm.

Or

(b) Solve the linear Diophantine equation $172x + 20y = 1000$.

18. (a) State and prove the fundamental theorem of arithmetic.

Or

(b) If all the $n > 2$ terms of the arithmetic progression $p, p+d, p+2d, \ldots, p(n-1)d$ are prime numbers, then show that the common difference d is divisible by every prime $q < n$.

19. (a) State and prove the Chinese remainder theorem.

Or

(b) (i) Prove that 41 divides $2^{20} - 1$

(ii) Obtain the remainder when $1! + 2! + 3! + \ldots + 100!$ is divided by 12.

20. (a) State and prove Wilson's theorem.

Or

(b) Let p be an odd prime prove that quadratic congruence $x^2 + 1 \equiv 0 (md\ p)$ has a solution if and only if $p \equiv 1 \pmod 4$.

Reg. No. : ....

Code No. : 41161 E **Sub. Code : JMMA 63/ JMMC 63**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Sixth Semester

Mathematics/Mathematics with CA — Main

NUMBER THEORY

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} \ldots + (-1)^n \binom{n}{n} =$

(a) n (b) 0

(c) $(-2)^n$ (d) 2^n

2. $0! =$

(a) 0 (b) ∞

(c) 1 (d) None

3. When we divide the square of any odd integer, the remainder is ————

(a) 1 (b) 3

(c) 5 (d) 7

4. $\gcd(-5, 5) =$ ________

(a) 1 (b) −1

(c) −5 (d) 5

5. If P is a prime and p/ab, then ________

(a) p/a (b) p/b

(c) (a) or (b) (d) (a) and (b)

6. The number of prime numbers of the form $n^3 - 1$ is ________

(a) 1 (b) 0

(c) 7 (d) ∞

7. $-31 \equiv$ ________ $(\text{mod } 7)$

(a) 10 (b) 11

(c) −4 (d) −5

8. If $ca \equiv cb (\text{mod } n)$ and $\gcd(c, n) = 1$, then ________

(a) $a \equiv b (\text{mod } n)$ (b) $a \equiv c (\text{mod } n)$

(c) $b \equiv c (\text{mod } n)$ (d) $a \equiv b \left(\text{mod } \frac{n}{c} \right)$

9. If p is a prime and a is any integer, then $a^p \equiv$ ________ $(\text{mod } p)$

(a) 1 (b) 0

(c) a (d) p

10. $12! + \equiv$ ________ $(\text{mod } 13)$

(a) −1 (b) 1

(c) 12 (d) 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 250 words.

11. (a) State and prove the Archimedean property.

Or

(b) If t_n is the n^{th} triangular number, prove that $t_1 + t_2 + \ldots + t_n = \frac{n(n+1)(n+2)}{6}, n \geq 1$

12. (a) (i) If $\gcd(a, b) = 1$, then show that $\left(\frac{a}{d}, \frac{b}{d} \right) = 1$.

(ii) State and prove the Euclid's lemma.

Or

(b) Prove : $\gcd(a, b) \times \text{lcm}(a, b) = ab$.

13. (a) Show that $\sqrt{2}$ is irrational number.

Or

(b) Prove that there are an infinite number of primes of the form $4n + 3$.

14. (a) Prove that $a \equiv b (\text{mod } n)$ if and only if a and b leave the same non negative remainder when divided by n.

Or

(b) Solve the system: $7x + 3y \equiv 10 (\text{mod } 16)$, $2x + 5y \equiv 9 (\text{mod } 16)$.

15. (a) Show that the converse of Fermat's theorem is not true.

Or

(b) If n is an odd pseudo prime, then show that $M_n = 2^n - 1$ is also an odd pseudo prime.