(6 pages) **Reg. No. :**..

Code No. : 20578 E **Sub. Code : SMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Sixth Semester

Mathematics — core

COMPLEX ANALYSIS

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. At $z = 0$, the function $f(z) = |z|^2$ is ____________.

   (a) analytic (b) differentiable

   (c) not differentiable (d) not continuous

2. If $u = \frac{1}{2}\log(x^2 + y^2)$ is a harmonic function, then $u_{xx} + u_{yy} =$ ____________.

   (a) ∞ (b) $-\infty$

   (c) 1 (d) 0

3. If C is the circle $|z-2|=5$, then $\int_C \frac{dz}{z-3}$ = ____________.

(a) πi (b) $2\pi i$

(c) $\frac{\pi}{2} i$ (d) 0

4. If C is a circle $|z|=1$, then $\int_C \frac{e^z}{z} dz$ = ____________.

(a) πi (b) $\frac{\pi}{2} i$

(c) $2\pi i$ (d) $2\pi i \cdot e$

5. The singular point of $\frac{1}{z}$ is ____________.

(a) $z = 0$ (b) $z = -1$

(c) $z = \infty$ (d) $z = 1$

6. The residue of $\cot z$ at $z = 0$ is ____________.

(a) 1 (b) -1

(c) 0 (d) ∞

7. If $f(z)=e^{1/2}$ then $z=0$ is __________.

(a) an essential singularity

(b) removable singularity

(c) a pole of order 2

(d) none of these

8. The order of pole '0' for $\frac{1-e^{2z}}{z^4}$ is __________.

(a) 0 (b) 1

(c) 2 (d) 3

9. The transformation $w=z+\alpha$ is a __________.

(a) reflection

(b) translation

(c) rotation

(d) none of these

10. The fixed point of the transformation $w=\frac{1}{z-2i}$ is

(a) i (b) $3i$

(c) $1+i$ (d) $2i$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that $\lim_{z \to 2} \frac{z^2 - 4}{z - 2} = 4$.

Or

(b) Show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2$.

12. (a) Evaluate $\int_C \frac{z dz}{z^2 - 1}$, C is the positively oriented circle $|z| = 2$.

Or

(b) State and prove Morere's Theorem.

13. (a) State and prove Rouche's Theorem.

Or

(b) Expand $\frac{-1}{(z-1)(z-2)}$ as a power series in z valid in $1 < |z| < 2$.

14. (a) Evaluate $\int_C \tan z dz$ where C is $|z| = 2$.

Or

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$.

15. (a) Prove that the cross ratio is preserved by a Bilinear transformation.

Or

(b) Find the image of the strip $2 < x < 3$ under $w = \frac{1}{z}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive C-R equations in polar co-ordinates.

Or

(b) Find the analytic function $f(z) = u + iv$, for which $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$.

17. (a) State and prove Cauchys integral formula.

Or

(b) State and prove Taylor's Theorem.

18. (a) Find the residue of $\frac{1}{z - \sin z}$ at it pole.

Or

(b) State and prove residue theorem.

19. (a) Prove that $\int_0^{2\pi} \frac{d\theta}{1 + a \sin\theta} = \frac{2\pi}{\sqrt{1 - a^2}}$, $(-1 < a < 1)$.

Or

(b) Prove that $\int_0^{\infty} \frac{\cos x}{1 + x^2} dx = \frac{\pi}{2e}$.

20. (a) Prove that any bilinear transformation can be expressed as product of translation, rotation, magnification and inversion.

Or

(b) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto $w_1 = 1, w_2 = i, w_3 = -1$ respectively.
