(7 pages) **Reg. No. :** ....

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M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2024

Fourth Semester

Mathematics — Core

TOPOLOGY — II

(For those who joined in July 2021 – 2022)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A space that has a countable basis at each of its points is said to be ————

   (a) Second countable (b) First countable

   (c) Hausdorff (d) Dense

2. A subset A of a space X is said to be dense if ————.

   (a) $A = X$ (b) $A = \overline{A}$

   (c) $A = \phi$ (d) $\overline{A} = X$

3. If every subspace of X is normal then the space X is ————

   (a) Completely normal

   (b) Lindelöf

   (c) Locally compact

   (d) Regular

4. A closed subspace of a normal space is ————

   (a) Regular (b) Compact

   (c) Normal (d) Dense

5. A compact Hausdorff space X is metrizable if and if X has ————.

   (a) Countable basis (b) Uncountable basis

   (c) Covers (d) Closed sets

6. A space X is ———— if each point x of X has a neighborhood that is metrizable in the subspace topology.

   (a) Countable basis (b) Locally metrizable

   (c) Homeomorphic (d) Complete

7. The collection $\mathcal{A} = \left\{(0, \frac{1}{n}) \mid n \in Z_+\right\}$ is ———— in (0, 1) but not in $\mathbb{R}$

(a) Locally finite (b) Metrizable

(c) Refinement (d) Countably finite

8. A collection $\mathcal{A}$ of subsets of X has the countable intersection property if every countable intersection of elements of $\mathcal{A}$ is ————.

(a) Empty (b) Constant 0

(c) Nonempty (d) Infinite

9. A Space X is a ———— if and only if every non empty open set in X is of the second category.

(a) Baire space (b) Empty interior

(c) Hausdorff space (d) Lindelöf

10. If A contains no open set of X other than the empty set then A has ————.

(a) Interior (b) Empty

(c) Empty interior (d) Second category

PART B — (5 × 5 = 25 marks)s

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that if X is normal, every pair of disjoint closed sets have neighborhoods whose closures are disjoint.

Or

(b) Let X be a topological space in which one point sets are closed prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subseteq U$.

12. (a) Prove that every compact Hausdorff space is normal.

Or

(b) Prove that a subspace of a completely regular space is completely regular and product of completely regular space is completely regular space is completely regular.

13. (a) State and prove imbedding theorem.

Or

(b) Give an example that a Hausdorff space with a countable basis need not be metrizable.

14. (a) Let X be a set $\mathfrak{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Then prove that

(i) Any finite intersection of elements of $\mathfrak{D}$ is an element of $\mathfrak{D}$.

(ii) If A is a subset of X that intersects every element of $\mathfrak{D}$ then A is an element of $\mathfrak{D}$.

Or

(b) Let $\mathcal{A}$ be a locally finite collection of subsets of X then prove

(i) The collection $B = \{\overline{A}\}_{A \in \mathcal{A}}$ of the closures of the elements of $\mathcal{A}$ is locally finite.

(ii) $\overline{\bigcup_{A \in \mathcal{A}} A} = \bigcup_{A \in \mathcal{A}} \overline{A}$.

15. (a) Prove that any open subspace Y of a Baire space X is itself a Baire space.

Or

(b) Show that every locally compact Hausdorff space is a baire space.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that a subspace and product of Hausdorff space is Hausdorff. Also prove that a subspace and product of a regular space is regular.

Or

(b) Give an example for a space that is Hausdorff but not regular. Also prove that the space R_e is normal.

17. (a) State and prove Urysohn lemma.

Or

(b) Prove that every regular space with a countable basis is normal.

18. (a) State and prove Tietze extension theorem.

Or

(b) Prove that every regular space X with a countable basis is metrizable.

19. (a) Let X be a metrizable space. If A is an open covering of X, then prove that there is an open covering $\in$ of X refining A that is countably locally finite.

Or

(b) Prove that an arbitrary product of compact spaces is compact in the product topology.

20. (a) Let X be a space let (y,d) be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions such that $f_n(x) \to f(x)$ for all $x \in X$. Where $f : X \to Y$. Prove that if X is a Baire space then the set of points at which f is continuous is dense in X.

Or

(b) State and prove Baire category theorem.
