(6 pages) **Reg. No. :** ....

Code No. : 7122 **Sub. Code : PMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2018.

Second Semester

Mathematics

DIFFERENTIAL GEOMETRY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The length of one complete turn of the helix $\bar{r} = (a \cos u, a \sin u, bu)$ is

(a) πab (b) $2\pi ab$

(c) $2\pi\sqrt{a^2+b^2}$ (d) $\pi a^2 b^2$

2. The normal in a direction orthogonal to the osculating plane is ———— line.

(a) tangent (b) normal

(c) binormal (d) none

3. The osculating circle at a point P on a curve has ———— point with the curve at P.

(a) one (b) two

(c) three (d) four

4. Radius of curvature ρ

(a) K (b) $\frac{1}{K}$

(c) τ (d) $\frac{1}{\tau}$

5. Since $\bar{r}_1 \times \bar{r}_2 \neq \bar{0}$ at an ordinary point the parametric curves $u = c_1$ and $v = c_2$ ———— each other.

(a) touch (b) cannot touch

(c) intersect (d) none

6. The parametric curves are orthogonal if

(a) $E = 0$ (b) $F = 0$

(c) $G = 0$ (d) $H = 0$

7. A sufficient condition for the parametric curve $u = \text{costant}$ to be geodesic is

(a) $U - V = 0$ (b) $U + V = 0$

(c) $U = 0$ (d) $V = 0$

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8. For the geodesic curvature vector $\overline{K}_g = \lambda \bar{r}_1 + \mu \bar{r}_2$, we have $\lambda =$

(a) $\frac{EV - FU}{H^2}$ (b) $\frac{UG - VF}{H^2}$

(c) $\frac{EG - F^2}{H^2}$ (d) $\frac{E^2 - FG}{H^2}$

9. Which of the following is correct?

(a) $\frac{\partial T}{\partial \dot{u}} = \bar{r} \cdot \dot{\bar{r}}_1$ (b) $T = \frac{1}{2}\dot{\bar{r}}^2$

(c) $\frac{\partial T}{\partial \dot{v}} = \bar{r} \cdot \dot{\bar{r}}_2$ (d) $U(t) = \bar{r} \cdot \ddot{\bar{r}}_1$

10. The point where $\frac{E}{L} = \frac{F}{M} = \frac{G}{N}$ is called __________.

(a) the centre (b) the focus

(c) the origin (d) an umbilic

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Derive Serret-Frenet formulae.

Or

(b) Prove that $K = \frac{|\dot{\bar{r}} \cdot \ddot{\bar{r}}|}{|\dot{\bar{r}}|^3}$ and $T = \frac{[\dot{\bar{r}}, \ddot{\bar{r}}, \dddot{\bar{r}}]}{|\dot{\bar{r}} \times \ddot{\bar{r}}|^2}$.

12. (a) Derive the locus of centre of spherical curvature.

Or

(b) Show that the ratio of the curvature to the torsion is constant at all points on a helix.

13. (a) Show that a proper transformation transforms a regular point into a regular point.

Or

(b) Compute the first fundamental coefficients E, F, G, H for this paraboloid $\bar{r} = (u, v, u^2 - v^2)$.

14. (a) Show that the curve $v =$ constant is a geodesic if and only if $EE_2 + FE_1 - 2EF_1 = 0$ for all values of u.

Or

(b) Prove that a necessary and sufficient condition for a curve $u = u(t)$, $v = v(t)$ on the surface $\bar{r} = \bar{r}(u, v)$ to be a geodesic is $U\frac{\partial T}{\partial V} - V\frac{\partial T}{\partial \dot{u}} = 0$.

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15. (a) If U, V are the intrinsic quantities of a surface at a point (u, v), prove that

(i) $K_g = \frac{1}{H}\left(V(s)\frac{\partial T}{\partial u'} - U(s)\frac{\partial T}{\partial V'}\right)$

(ii) $K_g = \frac{1}{H}\frac{U(s)}{u'}$ and (iii) $K_g = -\frac{1}{H}\frac{V(s)}{V'}$.

Or

(b) State and prove Meusnier's theorem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Derive the curvature and torsion of the curve of intersection of the quadratic surfaces $ax^2 + by^2 + cz^2 = 1$, $a^1x^2 + b^1y^2 + c^1z^2 = 1$.

Or

(b) Find the equation of the osculating plane at a general point on the cubic curve $\bar{r} = (u, u^2, u^3)$ and show that the osculating planes at any three points of the curve meet at a point lying in the plane determined by these three points.

17. (a) Derive the equation of an evolute of a curve.

Or

(b) Show that the torsion of an involuts of a curve is equal to $\frac{\rho(\sigma\rho^1 - \sigma^1\rho)}{(\rho^2 + \sigma^2)(c - s)}$.

18. (a) For the anchor ring $\bar{r} = ((b + a\cos u)\cos V, (b + a\cos u)\sin V\ a\sin u)$ compute E, F, G, H and prove that the area of the surface $(0, \le u \le 2\pi, 0 \le V \le 2\pi)$ is $4\pi^2 ab$.

Or

(b) Prove that the metric is invariant under a parametric transformation but the coefficients E, F, G are not invariant.

19. (a) State and prove the normal property of geodesics.

Or

(b) Find the geodesics on the surface with metric $ds^2 = e^{2r}(du^2 + U^2dV^2)$ whose U is a function of u alone.

20. (a) State and prove Rodrigus's formula.

Or

(b) If $K_1, K_2, \ldots K_m$ denotes the normal curvatures of m sections of a surface which make equal angles $\frac{2\pi}{m}$ with one another, prove that if $m > 2$, $K_1 + K_2 + \ldots + K_m = m_\mu$.

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