(7 pages) **Reg. No. :** ....

Code No. : 41319 E **Sub. Code : SMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2018.

First Semester

Mathematics – Main

CALCULUS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The radius of curvature of the curve $y = 4\sin x$ at the point $x = \frac{\pi}{2}$ is ————.

   (a) $\sqrt{2}$ (b) 2

   (c) $\frac{1}{\sqrt{2}}$ (d) $2\sqrt{2}$

2. If the radius of a circle is r, then its radius of curvature is ————.

   (a) r (b) $\frac{1}{r}$

   (c) r^2 (d) $\frac{1}{r^2}$

3. The evolute of the Cycloid $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$ is ————.

   (a) a circle (b) a straight line

   (c) another cycloid (d) catenary

4. The asymptote of the curve $r\theta = a$ is ————.

   (a) $r\sin\theta = a$ (b) $r\sin\theta = -a$

   (c) $r\cos\theta = a$ (d) $r\sin\theta = \frac{1}{a}$

5. A point is a node if $\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$ ———— $\frac{\partial^2 f}{\partial x^2}\cdot\frac{\partial^2 f}{\partial y^2}$.

   (a) $<$ (b) $>$

   (c) $=$ (d) $\neq$

6. The curve $y^2(1+x)=x^2(1-x)$ is symmetrical about ————.

(a) $x-$axis (b) y – axis

(c) both the axis (d) $y=x$

7. The value of $\int_0^a\int_0^b\int_0^c dx\,dy\,dz =$ ————.

(a) abc (b) $\frac{1}{abc}$

(c) $\frac{abc}{2}$ (d) $2\,abc$

8. If $x+y=u,\ y=uv$, then $J\left(\frac{u,v}{x,y}\right)=$ ————.

(a) v (b) u

(c) $\frac{1}{u}$ (d) $\frac{1}{v}$

9. The value of $\int_0^{\frac{\pi}{2}} \sin^6 x.\cos^5 x\,dx =$ ————.

(a) $\frac{8}{693}$ (b) $\frac{3\pi}{693}$

(c) $\frac{3\pi}{512}$ (d) $\frac{\pi}{693}$

10. The value of $\Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{p+1}{2}\right)$ ————.

(a) $\frac{\sqrt{\pi}}{2^{p-1}}\Gamma(p)$ (b) $\frac{\sqrt{\pi}}{2^{p+1}}\Gamma(p)$

(c) $\frac{\sqrt{\pi}}{2^{p}}\Gamma(p)$ (d) $\frac{\sqrt{\pi}}{2^{p+1}}\Gamma(p+1)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the radius of curvature of the curve $x^4+y^4=2$ at the point (1, 1).

Or

(b) Find the radius of curvature at any point on the curve $r=ae^{\theta\cot\alpha}$, where a and α are constants.

12. (a) Find the $p-r$ equation of the curve $r=\frac{a}{2}(1-\cos\theta)$.

Or

(b) Find the asymptotes of $y^3-6xy^2+11x^2y-6x^3+x+y=0$.

13. (a) Find the nature of the singular points on the curve, $(x+y)^3 = \sqrt{2}(y-x+2)^2$.

Or

(b) Show that, $x^4 - 2x^2y - xy^2 - 2x^2 - 2xy + y^2 - x + 2y + 1 = 0$ has a single cusp of the second kind at (0, 1).

14. (a) Evaluate $\iint (x^2+y^2)\,dx\,dy$ over the region $x \geq 0,\ y \geq 0$ and $x + y \leq 1$.

Or

(b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz\,dy\,dx}{(x+y+z+1)^3}$.

15. (a) Prove that $\Gamma(n+1) = n!$.

Or

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\,d\theta$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at the point (2, 1).

Or

(b) Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $x = y = \frac{3a}{2}$.

17. (a) Find the evaluate of the curve $x = a\cos^3\theta$; $y = a\sin^3\theta$.

Or

(b) Find the asymptotes of

$$x^3 + 2x^2y - 4xy^2 - 8y^3 - 4x + 8y = 1.$$

18. (a) Show that, $x^4 - 2x^2y - xy^2 - 2x^2 - 2xy + y^2 - x + 2y + 1 = 0$ has a single cusp of the second kind at $(0,-1)$.

Or

(b) Trace the curve whose equation is $y = \frac{x^2+1}{x^2-1}$.

19. (a) Evaluate $\iint (a^2 - x^2)\,dx\,dy$ taken over the half of the circle $x^2 + y^2 = a^2$ in the positive quadrant.

Or

(b) Prove that, $\frac{\partial(u,vw)}{\partial(x,y,z)} \cdot \frac{\partial(x,y,z)}{\partial(\xi,\eta,\lambda)} = \frac{\partial(u,vw)}{\partial(\xi,\eta,\gamma)}$.

20. (a) Prove that, $\frac{\beta(p, q+1)}{q} = \frac{\beta(p+1, q)}{p} = \frac{\beta(p, q)}{p+q}$.

Or

(b) Prove that, $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}}\,d\theta \int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,d\theta = \pi$.