(8 pages) **Reg. No. :** ....

Code No. : 20380 E **Sub. Code : CMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Third Semester

Mathematics — Core

SEQUENCES AND SERIES

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The following statements are true except

   (a) $\left(\frac{1}{n}\right)$ is a convergent sequence

   (b) $\left(\frac{1}{n}\right)$ is a bounded sequence

   (c) $\left(\frac{1}{n}\right)$ is a monotonic increasing sequence

   (d) $\left(\frac{1}{n}\right)$ is a strictly mono

2. Read the following statements

   (i) Any convergent sequence is a Cauchy sequence

   (ii) Any Cauchy sequence is a convergent sequence

   (iii) Any Cauchy sequence is a bounded sequence

   (iv) Any bounded sequence is a Cauchy sequence

   The correct statement

   (a) only (i) and (iii) are true

   (b) only (ii) and (iv) are true

   (c) (i), (ii), (iii) and (iv) are true

   (d) only (i) is true

3. The incorrect statement from the following (K_1, K_2)

   (a) $1+2+3+4+\cdots$ diverges to ∞

   (b) $\sum_{1}^{\infty}\left(\frac{1}{2^n}\right)$ converges to 1

   (c) $\sum_{1}^{\infty}\left(\frac{1}{3^n}\right)$ converges to $\frac{1}{2}$

   (d) $\sum_{1}^{\infty}\left(\frac{1}{n}\right)$ converges to 2

4. (i) The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p<1$

(ii) The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p>1$

The correct statement is ————

(a) only (i) is false

(b) only (ii) is false

(c) both (i) and (ii) are false

(d) both (i) and (ii) are true

5. $1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\cdots=$ ————

(a) 2 (b) –2

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

6. $\underset{n\to\infty}{lt}\frac{1}{n}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right)=$ ————

(a) 0 (b) e

(c) 1 (d) None

7. Let Σa_n be a series of positive terms. The correct statement from the following is

(a) Σa_n converges if $\underset{n\to\infty}{lt}\frac{a_n}{a_{n+1}}>1$

(b) Σa_n converges if $\underset{n\to\infty}{lt}\frac{a_n}{a_{n+1}}<1$

(c) Σa_n converges if $\underset{n\to\infty}{lt}\frac{a_{n+1}}{a_n}>1$

(d) Σa_n converges if $\underset{n\to\infty}{lt}\frac{a_{n+1}}{a_n}>0$

8. Applying the ratio test for $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}+\cdots$ the series is

(a) convergent

(b) divergent

(c) neither convergent nor divergent

(d) both convergent and divergent

9. $\underset{n\to\infty}{lt}\left(1+\frac{1}{1!}+\frac{1}{2!}+\cdots+\frac{1}{n!}\right)=$ ————

(a) 0 (b) 1

(c) e (d) ∞

10. $\lim_{n\to\infty} \frac{(1^3+2^3+\cdots+n^3)}{n^4} =$ ________

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{1}{4}$ (d) 0

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Show that a sequence cannot converge to two different limits.

Or

(b) Prove that if Σa_n converges and Σb_n diverges then $\Sigma(a_n+b_n)$ diverges.

12. (a) If $(a_n)\to a$ and $(b_n)\to b$ prove that $(a_n b_n)\to ab$.

Or

(b) Test the convergence of the Geometric series $1+r+r^2+\cdots+r^n+\cdots$ when

(i) $0 \le r \le 1$

(ii) $r > 1$

(iii) $r = 1$.

13. (a) Discuss the convergence of the series $\sum \frac{1}{\sqrt{n^3+1}}$.

Or

(b) If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ prove that $x = \frac{y}{1!} + \frac{y^2}{2!} + \frac{y^3}{3!} + \cdots$.

14. (a) Test the convergence of $\Sigma \frac{n^n}{n!}$.

Or

(b) Test the convergence of $\sum \sqrt{\frac{n}{n+1}}\,.x^n$.

15. (a) Test the convergence of $\Sigma \frac{(-1)^n \sin n\alpha}{n^3}$.

Or

(b) State and prove Dirichlet's test.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Show that the sequence $\left(1+\frac{1}{n}\right)^n$ converges.

Or

(b) If $(a_n) \to a$ and $a_n \neq 0$ for all n and $a \neq 0$ then prove that $\left(\frac{1}{a_n}\right) \to \frac{1}{a}$. Also prove $\left(\frac{a_n}{b_n}\right) \to \frac{a}{b}$ if $(a_n) \to a, (b_n) \to b$ where $b_n \neq 0$ for all n and $b \neq 0$.

17. (a) Applying Cauchy's general principle of convergence prove that $1 - \frac{1}{2} + \frac{1}{3} - \cdots + (-1)^n \frac{1}{n} + \cdots$ is convergent.

Or

(b) Show that the harmonic series $\Sigma \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

18. (a) State and prove comparison test.

Or

(b) State and prove Kummer's test.

19. (a) Test the convergence of the series $1 + \frac{\alpha\beta}{r} x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{r(r+1)2!} x^2 + \cdots$.

Or

(b) Test the convergence and divergence of the series $1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \frac{5^4x^4}{5!} + \cdots$.

20. (a) State and prove Cauchy's condensation test.

Or

(b) Test the convergence of the series $\sum (-1)^n \left(\sqrt{n^2+1} - n\right)$.

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