9. (a) State and prove Beurling's theorem.

Or

(b) If $0 < p \le \infty$, $f \in H^p$, and f is not identically 0, then at almost all points of T we have $f^*(e^{i\theta}) \ne 0$.

10. (a) Suppose A and C are positive constants and f is an entire function such that $|f(z)| \le C e^{A|z|}$ for all z and $\int_{-\infty}^{\infty} |f(x)|^2 dx < \infty$. Then there exists on $F \in L^2(-A, A)$ such that $f(z) = \int_{-A}^{A} F(t) e^{itz} dt$.

Or

(b) Find $\lim_{A \to \infty} \int_{-A}^{A} \frac{\sin \lambda t}{t} e^{itx} dt$ $(-\infty < x < \infty)$ where t is a positive constant.

Reg. No. : ....

Code No. : 9024 **Sub. Code : PMAC 12**

M.Phil. DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Mathematics – Main

ADVANCED ANALYSIS

(For those who joined in July 2018-2019 onwards)

Time : Three hours Maximum : 75 marks

PART A — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

1. (a) Let $\{E_k\}$ be a sequence of measurable sets in X, such that $\sum_{k=1}^{\infty} \mu(E_k) < \infty$. Then almost all $x \in X$ lie in at most finitely many of the sets E_k.

Or

(b) Define the following

(i) σ-algebra

(ii) measurable space.

2. (a) Show that there are uncountable sets $E \subset R'$ with $m(E) = 0$.

Or

(b) Suppose K is compact and F is closed, in a topological space X

3. (a) State and prove the Hahn Decomposition theorem.

Or

(b) If μ is a complex measure on X, then $|\mu|(X) < \infty$.

4. (a) If u is subharmonic in Ω, and if ϕ is a monotonically increasing convex function on R', then $\phi \circ u$ is subharmonic.

Or

(b) Suppose M_j is the inner factor of a function $f \in H^2$ and y is the smallest closed S-invariant subspace of H^2 which contain f. Then $y = M_j H^2$.

5. (a) Prove that each $c\{M_n\}$ is an algebra, with respect to pointwise multiplication.

Or

(b) If ϕ is a complex homomorphism on a Banach algebra A, then the norm of ϕ, as a linear functional, is a at most 1.

PART B — ($5 \times 10 = 50$ marks)

Answer ALL questions, choosing either (a) or (b).

6. (a) Suppose f and $g \in L'(\mu)$ and α and β are complex numbers. Then $\alpha f + \beta g \in L'(\mu)$, and $\int_X (\alpha f + \beta g) d\mu = \alpha \int_X f d\mu + \beta \int_X g d\mu$.

Or

(b) State and prove Lebesgue's monotone convergence theorem.

7. (a) State and prove Urysohn's Lemma.

Or

(b) State and prove the Vitali – Carathedory theorem.

8. (a) Prove that the total variation $|\mu|$ of a complex measure μ on M is a positive measure on M.

Or

(b) Prove that to each bounded linear functional ϕ on $Co(X)$, where X is a locally compact Hausdorff space, there corresponds a unique complex regular Borel measure μ such that $\phi(f) = \int_X f d\mu \ (f \in Co(X))$.