(8 pages) **Reg. No. :** ....

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M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2023.

Fourth Semester

Mathematics – Core

TOPOLOGY – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A space which contains a countable dense subset is called

(a) Separable

(b) Lindelöf

(c) Second countable

(d) Compact

2. Another name for Regular space is

(a) T_4 (b) $T_{2\frac{1}{2}}$

(c) $T_{3\frac{1}{2}}$ (d) T_3

3. Every regular Lindeloff space is

(a) normal

(b) completely regular but not normal

(c) regular but not completely regular

(d) compact and Hausdorff.

4. A space X is completely regular then it is homeomorphic to a subspace of

(a) $[0, 1]^J$

(b) $\mathbb{R}^n$ where n is a finite

(c) $\mathbb{R}^J$

(d) $(0, 1)^J$ where J is uncountable

5. Tietze extension theorem implies

(a) The Urysohn Metrization theorem

(b) Heine- Borel Theorem

(c) The Urysohn lemma

(d) The Tychonof theorem.

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6. Find the correct answer

(a) Subspace of a Normal space is normal

(b) Product of Normal spaces is normal

(c) R_ℓ^2 is completely regular

(d) R_K is regular but not normal

7. The set ———— is locally finite in R?

(a) $\{(n-1, x+1) : n \in Z\}$

(b) $\left\{\left(0, \frac{1}{n}\right) : n \in Z_+\right\}$

(c) $\left\{\left(\frac{1}{n+1}, \frac{1}{n}\right) : n \in Z_+\right\}$

(d) $\{(x, x+1) : x \in R\}$

8. Let $\mathcal{A} = \{(n-1, n+1) : n \in Z\}$. Which of the following refine $\mathcal{A}$.

(a) $\left\{\left(n-\frac{1}{2}, n+\frac{3}{2}\right) : n \in Z_+\right\}$

(b) $\left\{\left(n+\frac{1}{2}, n+\frac{3}{2}\right) : n \in Z_+\right\}$

(c) $\left\{\left(n-\frac{1}{2}, n+2\right) : n \in Z_+\right\}$

(d) $\{(x, x+1) : x \in R\}$

9. Which of the following is not true

(a) Every non empty open subset of the set of irrational numbers is of first category

(b) Open subspace of a Baire space is a Baire space

(c) Rationals as a subspace of real numbers is not a Baire space.

(d) If $X = \bigcup_{n=1}^{\infty} B_n$ and X is a Baire space with $B_1 \neq \phi$, then atleast one of $\overline{B_n}$ has nonempty interior.

10. Find the incorrect statement

(a) Any set X with discrete topology is a Baire space

(b) Every locally compact space is a Baire space

(c) [0, 1] is a Baire space

(d) The set of irrationals is not a Baire space

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PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let X be a space with one point sets in X are closed. Prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$.

Or

(b) Define $\mathbb{R}_k$ topological space. Prove that $\mathbb{R}_k$ is Hausdorff but not regular.

12. (a) Examine the proof of Urysohn lemma and show that for a given r, $f^{-1}(r) = \left(\bigcap_{p>r} U_p - \bigcup_{q<r} U_q \right)$, where p and q are rational.

Or

(b) Prove that every normal space is completely regular and completely regular space is regular.

13. (a) State and prove imbedding theorem.

Or

(b) Prove that Urysohn lemma can be proved by using Tietze extension theorem.

14. (a) Let A be a locally finite collection of subsets of X. Then prove that (i) The collection $B = \{\overline{A} : A \in \mathcal{A}\}$ is locally finite. (ii) $\overline{\bigcup_{A\in \mathcal{A}} A} = \bigcup_{A\in \mathcal{A}} \overline{A}$.

Or

(b) Define finite intersection property. Let X be a set and D be the set of all subsets of X that is maximal with respect to finite intersection property. Show that (i) $x \in \overline{A} \forall A \in D$ if and only if every neighborhood of x belongs to D. (ii) Let $A \in D$. Then prove that $B \supset A \Rightarrow B \in D$.

15. (a) Define a first category space. Prove that X is a Baire space if and only if 'given any countable collection $\{U_n\}$ of open sets in X, U_n is dense in $X \forall n$, then $\cap U_n$ is also dense'.

Or

(b) Define a Baire space. Whether Q the set of rationals as a space is a Baire space? What about if we consider Q as a subspace of real numbers space. Justify your answer.

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PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) What are the countability axioms. Prove that the space $\mathbb{R}_L$ satisfies all the countability axioms but the second.

Or

(b) Prove that product of Lindelof spaces need not be Lindelof.

17. (a) Define a regular space, a Lineloff space and a normal space. Prove that every regular Lindeloff space is normal.

Or

(b) (i) Prove that every normal space is completely regular and completely regular space is regular.

(ii) Prove that product of completely regular spaces is completely regular.

18. (a) State and prove Tietze extension theorem.

Or

(b) State and prove Uryzohn's metrization theorem.

19. (a) Let X be a metrizable space. If A is an open covering of X, then prove that there is an open covering ξ of X refining A that is countably locally finite.

Or

(b) State and prove Tychonoff theorem.

20. (a) Let X be a space; let (Y, d) be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions such that $f_n(x) \to f(x)$ for all $x \in X$, where $f : X \to Y$. If X is a Baire space, prove that the set of points at which f is continuous is dense in X.

Or

(b) State and prove Baire Category theorem.

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