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Reg. No. :

Code No. : 30344 E Sub. Code : SMMA 61

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Sixth Semester

Mathematics — Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. If $f(z) = z^2$, then the value of $v(x, y)$

- (a) $x^2 - y^2$ (b) $2xy$
(c) xy (d) $x^2 + y^2$

2. The complex form of CR equations

- (a) $f_x = -if_y$ (b) $f_x = if_y$
(c) $f_y = -if_x$ (d) $f_x = f_y$

3. If C is the circle with center a and radius r , then the value of $\int_C |z'(t)| dt$

- (a) $2\pi i$ (b) $-2\pi i$
(c) $2\pi r$ (d) $-2\pi r$

4. If C is the circle $|z - 2| = 5$, then $\int_C \frac{dz}{z - 3} = \text{---}$

- (a) 0 (b) $2\pi i$
(c) $-2\pi i$ (d) πi

5. $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots = ?$

- (a) $\sin z$ (b) $\cos z$
(c) $\sinh z$ (d) $\cosh z$

6. The poles of $f(z) = \frac{z^2}{(z - 2)(z + 3)}$

- (a) 2, 3 (b) -2, 3
(c) 2, -3 (d) -2, -3

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7. If $f(z) = \frac{1}{2z^2 + 5iz - 2}$, then $\text{Res}\{f(z); -i/2\} = ?$

- (a) $\frac{1}{3}$ (b) $\frac{1}{3i}$
(c) $-\frac{1}{3i}$ (d) $-\frac{1}{3}$

8. The value of $\int_{|z|=2} \tan z \, dz$

- (a) $2\pi i$ (b) $-2\pi i$
(c) $4\pi i$ (d) $-4\pi i$

9. The fixed point of the transformation $w = \frac{1}{z-2i}$

- (a) 0 (b) i
(c) $-i$ (d) $2i$

10. Which one of the following is not a bilinear transformation

- (a) $w = z$ (b) $w = \bar{z}$
(c) $w = 1 + z$ (d) $w = 1 - z$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $f(z) = \text{Re } z$ is nowhere differentiable.

Or

(b) If $f(z)$ and $\overline{f(z)}$ are analytic in a region D show that $f(z)$ is constant in that region.

12. (a) Prove that $\int_{-c} f(z) \, dz = - \int_c f(z) \, dz$.

Or

(b) Evaluate $\int_C \frac{z \, dz}{z^2 - 1}$ where C is the positively oriented circle $|z| = 2$.

13. (a) Expand $\cos z$ into a Taylor's series about the point $z = \pi/2$.

Or

(b) Find the residue of $\frac{1}{(z^2 + a^2)^2}$ at $z = ai$.



14. (a) Evaluate $\int_C \frac{dz}{2z+3}$ where C is $|z|=2$.

Or

- (b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

15. (a) Find the bilinear transformation which maps the point $z = -1, 1, \infty$ respectively on $w = -i, -1, i$.

Or

- (b) Find the fixed points of the transformation $w = \frac{1+z}{1-z}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Derive the CR equations in polar co-ordinates.

Or

- (b) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y)$.

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17. (a) State and prove Cauchy's integral formula.

Or

- (b) State and prove fundamental theorem of algebra.

18. (a) State and prove Maclaurin's series.

Or

- (b) State and prove Cauchy's residue theorem.

19. (a) Evaluate $\int_0^\infty \frac{x \sin x}{x^2 + a^2} dx$.

Or

- (b) Prove that $\int_0^\infty \frac{\cos x}{1+x^2} dx = \frac{\pi}{2e}$.

20. (a) Find the points where the following mappings are conformal. Also find the critical points if any (i) $w = z^n$ (ii) $w = \frac{1}{z}$.

Or

- (b) Prove that a bilinear transformation preserves inverse points.

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