(7 pages) **Reg. No. :** .....

Code No. : 20656 E **Sub. Code : ESMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

First Semester

Mathematics

Skill Enhancement Course — MATHEMATICS FOR COMPETITIVE EXAMINATIONS – I

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions by choosing the correct answer.

1. $\dfrac{(846+268)^2+(846-268)^2}{846\times846+268\times268}$ = _____.

(a) 846 (b) 268

(c) 2 (d) 1

2. What is the average of all prime numbers between 30 and 50?

(a) 39.8 (b) 35

(c) 40 (d) 41

3. The sum of the squares of three numbers which are in the ratio 2 : 3 : 4 is 725. What are these numbers?

(a) 20, 15, 30 (b) 14, 21, 28

(c) 10, 15, 20 (d) 20, 30, 40

4. If $2A = 3B = 4C$, then $A : B : C$ = _____.

(a) 2 : 3 : 4 (b) 4 : 3 ; 2

(c) 6 : 5 : 3 (d) 6 : 4 : 3

5. Raman and Rajan started a business investing Rs. 22,500 and Rs. 35,000 respectively. Out of a total profit of Rs. 13,800, Deepak's share amount is _____.

(a) Rs. 5,400 (b) Rs. 8,400

(c) Rs. 7,200 (d) Rs. 9,600

6. Which is the largest in $16\frac{2}{3}\%, \frac{1}{3}, \frac{2}{15}$ and 0.18?

(a) $16\frac{2}{3}\%$ (b) $\frac{2}{15}$

(c) 0.18 (d) $\frac{1}{3}$

7. Find C.P when SP = Rs. 40.60 and gain = 16%

(a) Rs. 40 (b) Rs. 30

(c) Rs. 35 (d) Rs. 42

8. I gain 70 paise on Rs. 70/-. What is my gain percentage?

(a) 0.1% (b) 1%

(c) 7% (d) 10%

9. A number when divided by 6 is diminished by 40 find out the number.

(a) 72 (b) 84

(c) 60 (d) 48

10. If the sum of two numbers is 25 and their difference is 13, their product is ————————.

(a) 104 (b) 114

(c) 315 (d) 325

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $a^2 + b^2 = 117$ and $ab = 54$ find the value of $\frac{a+b}{a-b}$.

Or

(b) The average of three friends is 23. Even if the age of the fourth friend is added, the average remains 23. What is the age of the fourth friend?

12. (a) If $5x^2 - 13xy + 6y^2 = 0$, then find $x : y$.

Or

(b) The ratio of the incomes of A and B is 5 : 4 and the ratio of their expenditures is 3 : 2. If at the end of the year, each saves Rs. 1,600. What is the income of A?

13. (a) A, B, C hired a car for Rs. 520 and used it for 7, 8 and 11 hours respectively. Find the hire charges paid by B.

Or

(b) A, B, C hire a meadow for Rs. 1,460. A puts in 10 cows for 20 days $B - 30$ cows for 8 days and $C - 16$ cows for 9 days. Find the rent paid by each one of them.

14. (a) A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. Then how much quantity sold at 18% profit?

Or

(b) When a plot is sold for Rs. 18,700, the owner loses 15%. At what price must the plot be sold in order to gain 15%.

15. (a) If three numbers are added in pairs, the sums equal to 10, 19 and 21. Find the numbers.

Or

(b) The sum of a positive numbers and its reciprocal is thrice the difference of the number and its reciprocal. Find the number.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) The price of 10 chairs is equal to that of 4 tables. The price of 15 chairs and 2 tables together is Rs. 4,000. Find the total price of 12 chairs and 3 tables.

Or

(b) If $2x + 3y + z = 55$; $x + z - y = 4$ and $y - x + z = 12$ then find the values of x, y and z.

17. (a) 10 years ago, the average age of a family of four members was 24 years. Two children having been born (with age difference of two years), the present age of the family is the same. What is the present age of the youngest child?

Or

(b) Three numbers A, B and C are in the ratio $1:2:3$. Their average is 600. If A is increased by 10% and B is decreased by 20%, then to get the average increased by 5%. How much C will increase?

18. (a) The length of the rectangle is increased by 60%. By what percent would the width have to be decreased to maintain the same area?

Or

(b) 8% of the voters in an election did not cast their votes. In this election, there were only two candidates the winner by obtaining 48% of the total votes defeated his constant by 1100 votes. Find the total number of voters in the election.

19. (a) After getting two successive discounts, a shirt with a list price of Rs. 150 is available at Rs. 105. If the second discount is 12.5%m find the first discount.

Or

(b) Sankar purchased 20 dozen mangoes at Rs. 48 per dozen. He sold 8 dozen at 10% profit and the remaining 12 dozen with 20% profit. What is his profit percentage in the transaction?

20. (a) A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.

Or

(b) If $2\frac{1}{2}$ is added to a number and the sum multiplied by $4\frac{1}{2}$ and 3 is added to the product and then dividing the sum by $1\frac{1}{5}$, the quotient becomes 25, what is the number?

(7 pages)

Reg. No. : .....

Code No. : 20653 E **Sub. Code : EMMA 12**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

First Semester

Mathematics – Core

DIFFERENTIAL CALCULUS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $D^n(ax+b)^{-1}$ = _____.

(a) $(-1)^n a^n (ax+b)^{-n-1}$

(b) $(-1)^n n! a^n (ax+b)^{-n-1}$

(c) $(-1)^n a^n (ax+b)^{-n}$

(d) $(-1)^n n! a^n (ax+b)^{-n}$

2. $D^n(\cos x)$ = _____.

(a) $\cos\left(\frac{n\pi}{2}+x\right)$ (b) $\sin\left(\frac{n\pi}{2}+x\right)$

(c) $\cos\frac{n\pi}{2}x$ (d) $\sin\frac{n\pi}{2}x$

3. If $z=f(u)$ and $u=\varphi(x,y)$ (x,y are independent variables), then $\frac{\partial z}{\partial x}$ = _____.

(a) $\frac{\partial z}{\partial u}\frac{du}{dx}$ (b) $\frac{\partial z}{\partial u}\frac{du}{dy}$

(c) $\frac{dz}{du}\frac{\partial u}{\partial x}$ (d) $\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}$

4. If $x^3+y^3=3axy$, $\frac{dy}{dx}$ = _____.

(a) $\frac{x^2-ay}{y^2-ax}$ (b) $\frac{y^2-ax}{x^2-ay}$

(c) $\frac{ax-y^2}{x^2-ay}$ (d) $\frac{ay-x^2}{y^2-ax}$

5. $f(x,y)=\dfrac{x^3-y^3}{x+y}$ is a homogeneous function of degree _____.

(a) 2 (b) 4

(c) 3 (d) 1

6. If $f(x,y)$ is a homogeneous function of degree n, _____.

(a) $f(\lambda x, y)=\lambda^n f(x,y)$

(b) $f(x,\lambda y)=\lambda^n f(x,y)$

(c) $f(\lambda x,\lambda y)=\lambda^n f(x,y)$

(d) $f(\lambda x,\lambda y)=\lambda^{2n} f(x,y)$

7. The envelope of the family of curves $\dfrac{x\cos\alpha}{a}+\dfrac{y\sin\alpha}{b}=1$ where α is the parameter and a and b are constants is _____.

(a) a circle (b) an ellipse

(c) a straight line (d) a parabola

8. The evolute of a curve is the _____ of the normals to the curve.

(a) involute (b) evolute

(c) envelope (d) normal

9. The radius of curvature at the point $x=\pi/2$ on the curve $y=\sin x$ is _____.

(a) 1 (b) –1

(c) 0 (d) 2

10. The centre of curvature of the curve $xy=c^2$ at the point (c, c) is _____.

(a) (c, c) (b) $(2c, c)$

(c) $(c, 2c)$ (d) $(2c, 2c)$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $y=\log(ax+b)$, find y_n.

Or

(b) If $xy=ae^x+be^{-x}$, prove that $x\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}-xy=0$.

12. (a) If $u=\log\dfrac{x^2+y^2}{xy}$, prove that $\dfrac{\partial^2 u}{\partial x\partial y}=\dfrac{\partial^2 u}{\partial y\partial x}$.

Or

(b) Find $\dfrac{du}{dt}$ where $u=x^2+y^2+z^2$, $x=e^t$, $y=e^t\sin t$, $z=e^t\cos t$.

13. (a) Verify Euler's theorem for the function $u = x^3 - 2x^2y + 3xy^2 + y^3$.

Or

(b) If $u = \sin\left(\frac{x^2+y^2}{x+y}\right)$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{x^2+y^2}{x+y}\cos\left(\frac{x^2+y^2}{x+y}\right).$$

14. (a) Find the envelope of the family of circles $x^2 + y^2 - 2ax\cos\theta - 2ay\sin\theta = c^2$ (θ-parameter).

Or

(b) Find the envelope of the family of curves $y = m^2x + am$ (m-parameter).

15. (a) Find the radius of curvature of the curve $r = a(1 - \cos\theta)$.

Or

(b) Find the centre of curvature of the curve $y = x\log x$ at the point where $y_1 = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find (i) $D^n(\cos x\cos 2x\cos 3x)$

(ii) $D^n\left(\log\frac{2x+3}{3x+2}\right)$.

Or

(b) If $y = \sin(m\sin^{-1}x)$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

17. (a) If $V = \left(x^2+y^2+z^2\right)^{-1/2}$, prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$.

Or

(b) If $x = e^{-t}\cos\theta$, $y = e^{-t}\sin\theta$, prove that $\frac{\partial t}{\partial x} = \frac{-x}{x^2+y^2}$ and $\frac{\partial\theta}{\partial x} = \frac{-y}{x^2+y^2}$.

18. (a) State and prove Euler's theorem.

Or

(b) If $u = \tan^{-1}\left(y^2/x\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 2u\sin^2 u.$$

19. (a) Find the envelope of the family of straight lines $\frac{x}{a}+\frac{y}{b}=1$ where $a^2+b^2=k^2$ and k is a constant.

Or

(b) Prove that the envelope of the family of ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a^2+b^2=c^2)$ are $x+y=\pm c$ and $x-y=\pm c$.

20. (a) Find the radius of curvature at the point 't' of the curve $x=a(\cos t+t\sin t)$; $y=a(\sin t-t\cos t)$.

Or

(b) Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$; $y=a(1-\cos\theta)$ is another cycloid.
