(7 pages) **Reg. No. :** ....

Code No. : 41158 E **Sub. Code : JMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Sixth Semester

Mathematics — Main

LINEAR ALGEBRA

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The smallest subspace of V containing subspaces A and B is
   (a) $A \cap B$ (b) $A \cup B$
   (c) $A + B$ (d) $A \oplus B$

2. Which of the following is of a subspace of a vector space R^3?
   (a) $W = \{(a, 0, 0) / a \in R\}$
   (b) $W = \{(ka, kb, kc) / k \in R\}$
   (c) $W = \{(a, a+1, 0) / a \in R\}$
   (d) $W = \{(a, 0, b) / a, b \in R\}$

3. In $V_3(R)$, if $S = \{(1, 0, 0), (2, 0, 0), (3, 0, 0)\}$, then $L(S) =$ ——————
   (a) S
   (b) $\{(x, y, 0) / x, y \in R\}$
   (c) $\{(x, 0, 0) / x \in R\}$
   (d) $V_3(R)$

4. $\dim M_2(R) =$ ——————
   (a) 1 (b) 2
   (c) 3 (d) 4

5. For the linear transformation T, if rank $T = \dim V$, then ——————
   (a) T is 1 − 1 (b) T is not 1 − 1
   (c) nullity $T \neq 0$ (d) None

6. The norm of the vector (1, 2, 3) is ——————
   (a) 6 (b) 14
   (c) $\sqrt{14}$ (d) $\sqrt{6}$

7. The inverse of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

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(a) $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ (b) $\begin{pmatrix} -a & c \\ b & -d \end{pmatrix}$

(c) $\frac{1}{|A|} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ (d) $\frac{1}{|A|}\begin{pmatrix} -a & c \\ b & -d \end{pmatrix}$

8. The rank of the matrix $\begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ is

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(a) 1 (b) 2

(c) 3 (d) 4

9. The eigen values of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ are

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(a) $-1, 1$ (b) $-1, -1$

(c) $1, -1$ (d) $1, 1$

10. The sum of the eigen values of $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$ is ________________

(a) 0 (b) 1

(c) $2\cos\theta$ (d) $\cos^2\theta$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let V be a vector space over F. Show that a non empty subset W of V is a subspace of V if and only if W is closed with respect to vector addition and scalar multiplication in V.

Or

(b) Show that the union of two subspaces of a vector space is a subspace if and only if one is contained in the other.

12. (a) Prove that any set containing a linearly dependent set is also a linearly dependent set.

Or

(b) Let V be a finite dimensional vector space over a field F. Let A be a subspace of V. Then show that there exists a subspace B of V such that $V = A \oplus B$.

13. (a) Find the linear transformation $T : V_3(R) \to V_3(R)$ determined by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ with respect is the standard basis $\{e_1, e_2, e_3\}$.

Or

(b) Show that for a finite dimensional inner product space V and a subspace W of V, $V = W \oplus W^{\perp}$.

14. (a) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & -1 \\ -2 & 1 & 3 \end{pmatrix}$.

Or

(b) Show that the equations are consistent and solve them $x + y + z = 6$; $x + 2y + 3z = 14$; $x + 4y + 7z = 30$

15. (a) State and prove Cayley Hamilton theorem.

Or

(b) The produce of two eigen values of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16. Find all eigen values of A.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove fundamental theorem of homomorphism.

Or

(b) For vector space V and its subspace A and B, prove that $V = A \oplus B$ if and only if every element of V can be uniquely expressed in the form $a + b$ where $a \in A$ and $b \in B$.

17. (a) Let V be a finite dimensional vector space over a field F. Let W be a subspace of V. Then show that (i) $\dim W \leq \dim V$ (ii) $\dim \frac{V}{W} = \dim V - \dim W$.

Or

(b) Let V be a vector space over F. Let $S, T \subseteq V$, then prove that

(i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

(ii) $L(S \cup T) = L(S) + L(T)$

(iii) $L(S)$ is the smallest subspace containing S

(iv) $L(S) = S \Leftrightarrow S$ is a subspace of V.

18. (a) State and prove Schwartz's inequality and triangle inequality.

Or

(b) Show that every finite dimensional inner product space has an orthonormal basis.

19. (a) Find the rank of the matrix $A = \begin{pmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 7 \end{pmatrix}$.

Or

(b) For what values of λ and μ, the system of equations is consistent.

$x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$. Solve them.

20. (a) Using Cayley Hamilton's theorem. Find A^{-1} and A^4. $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$.

Or

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{pmatrix}$.
