(8 pages)

Reg. No. : ....

Code No. : 8158   Sub. Code : VPHC 11

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2024

First Semester

Physics – Core

MATHEMATICAL PHYSICS

(For those who joined in July 2024 onwards)

Time : Three hours   Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. The concept of linear vector space is important for the study of

   (a) Electronics

   (b) Quantum mechanics

   (c). Space science

   (d) None of the above

2. The operation of addition in liner vector space must satisfy

   (a) Commutative law

   (b) Associative law

   (c) Both commutative and Associative laws

   (d) None of the above

3. The Elements of vector space are

   (a) Vectors

   (b) Scalars

   (c) Both vectors and scalars

   (d) None of the above

4. If a group is cyclic, it must also be

   (a) Isomorphic   (b) Homomorphic

   (c) Abelian   (d) None of the above

5. If $z = x + iy$ and be a complex number, its modulus is denoted by $|z|$ and given by

   (a) $z = +\sqrt{x^2 + y^2}$ (b) $z = -\sqrt{x^2 + y^2}$

   (c) $z = +\sqrt{x^2 - y^2}$ (d) $z = -\sqrt{x^2 - y^2}$

6. Modulus of the product of two complex numbers is the product of their
   (a) Arguements
   (b) Moduli
   (c) Real numbers
   (d) None of the above

7. A matrix of the order of $q \times 1$ is called
   (a) Row matrix
   (b) Column matrix
   (c) Identity matrix
   (d) None of the above

8. Scalar quantity which is associated with a linear transformation belonging to a vector space is
   (a) Eigen matrix
   (b) Eigen matrix
   (c) Eigen Value
   (d) All the above

9. A matrix that has all of its elements is zero is said to be a
   (a) Unit matrix
   (b) Identify matrix
   (c) Row matrix
   (d) Null matrix

10. Which of the following represents Laplace's transform?
   (a) $g(\alpha) = \int_0^\infty f(t) e^{-i\alpha t} dt$

   (b) $g(\alpha) = \int_0^\infty f(t) e^{-\alpha t} dt$

   (c) $g(\alpha) = \int_0^\infty f(t) t J_n(\alpha t) dt$

   (d) $g(\alpha) = \int_0^\infty f(t) t^{\infty-1} dt$

11. The fourier sine transform of function $f(x) = e^{-ax}$ is
   (a) $\dfrac{s}{a^2 + s^2}$
   (b) $\dfrac{a}{a^2 + s^2}$

   (c) $\dfrac{\sin ax}{\sqrt{a^2 + s^2}}$
   (d) $\dfrac{a \sin ax}{\sqrt{a^2 + s^2}}$

12. Laplace transform of t^n is
   (a) $\dfrac{n!}{s^n}$
   (b) $\dfrac{n!}{s^{n-1}}$

   (c) $\dfrac{n!}{s^{n+1}}$
   (d) $\dfrac{(n+1)!}{g^{n+1}}$

13. The Legendre polynomial $P_n(x)$ has
   (a) n real zero between 0 and 1
   (b) n zero of which only one is between -1 and +1
   (c) $(2n-1)$ real zeros between -1 and +1
   (d) No real zero between 0 and z

14. In $P_n(x)$ is Legendre polynomial, then the value of
   $\int_{-1}^{+1} P_n(x) P_m(x) dx (m \neq n)$ is equal to
   (a) 0
   (b) 1
   (c) $\dfrac{2}{(2m+1)(2m-1)}$
   (d) $\dfrac{m! n!}{2(m+n)!}$

15. The orthogonality condition for legendre polynomial $\int_{-1}^{+1} P_m(x) P_n(x) dx = 0$ is true for

   (a) $m = n$ (b) $m \neq n$

   (c) $m = n = 0$ (d)   None of these

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) What is linear vector space? Describe orthogonality.

Or

   (b) What is dual space? Describe orthogonal basis.

17. (a) Describe differentiability.

Or

   (b) How the character tables are construed.

18. (a) Diagonalise the matrix $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Or

   (b) Find the rank of the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}$.

19. (a) State and prove similarity theorem.

Or

   (b) State and prove change of scale property of Laplace transforms.

20. (a) Solve the differential equation $\dfrac{d^2 y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 0$.

Or

   (b) Solve $(D^2 - 4D + 4)y = x^3 e^{2x}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) Explain Gram-Schmidt orthogonalization procedure.

Or

   (b) Explain orthogonal transformations and rotation of R^2 vector space with standard basis.

22. (a) Write down the properties of moduli and arguements.

Or

(b) Write down the algebric operations of complex number.

23. (a) State and prove Cayley Hamilton theorem.

Or

(b) Find the characteristic equations of a matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

24. (a) Find the Laplace transform of

(i) $sin\,h\,at$

(ii) $cosh\,at$

Or

(b) Write down the Fourier sine and cosine transform of first derivatives.

25. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} = +4\frac{dy}{dx} + 3y = 65\cos 2x.$$

Or

(b) Using Rodrigue's formula, prove that

(i) $\displaystyle\int_{-1}^{+1} P_o(x)\,dx = 2$

(ii) $\displaystyle\int_{-1}^{+1} P_n(x)\,dx = 0 \;\; (n \neq 0).$

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