

(7 pages)

Reg. No. :

Code No. : 7352

Sub. Code : HMAM 12

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2015.

First Semester

Mathematics

ANALYSIS — I

(For those who joined in July 2012 onwards)

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If E is closed and if every point of E is a limit point of E , then E is _____.
(a) open (b) bounded
(c) perfect (d) dense in X
2. The set of all integers is _____.
(a) closed (b) open
(c) perfect (d) bounded

3. If $s_n = 1 + \frac{(-1)^n}{n}$, the sequence $\{s_n\}$ _____.

- (a) converges to 0 (b) converges to 1
(c) diverges (d) is unbounded

4. If $p > 0$, then $\lim_{n \rightarrow \infty} \frac{1}{n^p} =$ _____.

- (a) 0 (b) 1
(c) ∞ (d) n

5. For the series, $\frac{1}{2} + 1 + \frac{1}{8} + \frac{1}{4} + \frac{1}{32} + \frac{1}{16} + \dots$,

$$\limsup_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} =$$

- (a) 0 (b) 1
(c) $\frac{1}{2}$ (d) 2

6. The radius of convergence R for the series $\sum \frac{z^n}{n^2}$ is _____.

- (a) 0 (b) 1
(c) 2 (d) $\frac{1}{2}$

Page 2

Code No. : 7352



7. The function

$$f(x) = \begin{cases} x, & x \text{ rational} \\ 0, & x \text{ irrational} \end{cases}$$

is continuous at _____.

- (a) every point
(b) every point other than 0
(c) $x = 0$
(d) every rational x
8. The number of points at which monotonic functions have discontinuities of the second kind is _____.
- (a) infinite (b) 1
(c) finite (d) 0
9. Let f be a real function defined on a metric space X . If there exists $\delta > 0$ such that $f(q) \leq f(p)$ for all $q \in X$ with $d(p, q) < \delta$, then we say that _____.
- (a) f is continuous at $p \in X$
(b) f has a local maximum at $p \in X$
(c) f has a local minimum at $p \in X$
(d) f is uniformly continuous at $p \in X$

10. If f is a real continuous function on $[a, b]$ which is differentiable in (a, b) , then there is a point $x \in (a, b)$ at which $f(b) - f(a) =$

- (a) $(b - a)f(x)$ (b) $(b - a)f'(x)$
(c) $f(x)$ (d) $f'(x)$

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Suppose $Y \subset X$. Prove that a subset E of Y is open relative to Y if and only if $E = Y \cap G$ for some open subset G of X .

Or

- (b) Define the cantor set P and prove that P is perfect.

12. (a) Suppose $\{s_n\}$ and $\{t_n\}$ are complex sequences and $\lim_{n \rightarrow \infty} s_n = s$, $\lim_{n \rightarrow \infty} t_n = t$. Then show that $\lim_{n \rightarrow \infty} s_n t_n = st$.

Or

- (b) Prove :

(i) If $p > 0$, then $\lim_{n \rightarrow \infty} \sqrt[n]{p} = 1$.

(ii) $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$.



13. (a) State and prove the Ratio test.

Or

- (b) For any sequence $\{c_n\}$ of positive numbers, show that $\liminf_{n \rightarrow \infty} \frac{c_{n+1}}{c_n} \leq \liminf_{n \rightarrow \infty} \sqrt[n]{c_n}$.

14. (a) Prove that continuous function of a continuous function is continuous.

Or

- (b) If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X , show that $f(E)$ is connected.

15. (a) On the segment $(0, 1)$, define $f(x) = x$ and $g(x) = x + x^2 e^{i/x^2}$. Prove that L-Hospital rule fails for this case.

Or

- (b) Let

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Find $f'(x)$.

Page 5

Code No. : 7352

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove :

- (i) Suppose $K \subset Y \subset X$. Then K is compact relative to X if and only if K is compact relative to Y .
(ii) Closed subsets of compact sets are compact.

Or

- (b) Prove that every K -cell is compact.

17. (a) Show that $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ and e is irrational.

Or

- (b) Prove :

- (i) Suppose $a_1 \geq a_2 \geq a_3 \geq \dots \geq 0$. Then the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the series $a_1 + 2a_2 + 4a_4 + 8a_8 + \dots$ converges.
(ii) $\text{diam } \overline{E} = \text{diam } E$.

18. (a) For any two real sequence $\{a_n\}$, $\{b_n\}$ prove that $\limsup_{n \rightarrow \infty} (a_n + b_n) \leq \limsup_{n \rightarrow \infty} a_n + \limsup_{n \rightarrow \infty} b_n$.

Or

Page 6

Code No. : 7352



(b). Find the radius of convergence of each of the following power series :

(i) $\sum n^3 z^n$

(ii) $\sum \frac{2^n}{n!} z^n$

(iii) $\sum \frac{2^n}{n^2} z^n$

(iv) $\sum \frac{n^3}{3^n} z^n$

19. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y . Prove that f is uniformly continuous on X .

Or

(b) Let E be a non-compact set in R^1 . Prove that

(i) there is a continuous function on E which is not bounded

(ii) there is a continuous and bounded function on E which has no maximum.

20. (a) State and prove the Chain rule.

Or

(b) State and prove L'Hospital's rule.

