

17. (a) Show that $H_{m,n}$ is m -connected.

Or

- (b) Prove that a nonempty connected graph is Eulerian if and only if no vertex of the graph is of odd degree.

18. (a) State and prove Vizing's theorem.

Or

- (b) Show that G has a perfect matching if and only if $|o(G-S)| \leq |S|$ for all $S \subset V$.

19. (a) If $\delta > 0$, then prove that $\alpha' + \beta' = v$.

Or

- (b) If $k \geq 2, l \geq 2$, prove that $r(k, l) \leq r(k, l-1) + r(k-1, l)$ and that, if both $r(k, l-1)$ and $r(k-1, l)$ are even, then strict inequality holds in the above.

20. (a) State and prove Dirac's Theorem on critical graphs having 2-vertex cut.

Or

- (b) State and prove Brooke's Theorem.

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M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2010.

Third Semester

Mathematics

GRAPH THEORY ITS APPLICATIONS

(For those who joined in July 2008 and afterwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

1. Define incidence matrix of a graph. Give an example.
2. Show that every connected graph contains a spanning tree.
3. If G is 2-connected, prove that any two vertices of G lie on a common cycle.
4. Define the graph $H_{m,n}$.
5. When a path is said to be augmented with respect to a matching?
6. Give an example with $\chi' = \Delta + \mu$.

7. Define maximal independent set in a graph. Give an example.
8. Define k -colourable graph. Give an example of a 3-colourable graph.
9. Define subdivision of a graph.
10. Define uniquely k -colourable graphs.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define the degree of a vertex in a graph and prove that the sum of the degrees of the vertices of a graph is twice the number of edges of the graph. Deduce that the number of vertices of a graph having odd degree is even.

Or

- (b) Show that a connected graph is a tree if and only if every edge is a cut edge.
12. (a) Prove that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.

Or

- (b) Construct a simple graph with
 - (i) $K=2$, $K'=4$ and $\delta=5$
 - (ii) $K=3$, $K'=4$ and $\delta=5$.

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13. (a) Prove that a matching M in G is a maximum matching if and only if G contains no M -augmenting path.

Or

- (b) Let G be a k -regular bipartite graph with $k > 0$, prove that G has a perfect matching.
14. (a) Define an independent set in a graph G and prove that S is independent if and only if $V-S$ is a covering of G .

Or

- (b) In a bipartite graph G with $\delta > 0$, prove that the number of vertices in a maximum independent set is equal to number of edges in a maximum edge covering.
15. (a) Define a k -critical graph and prove that for such a graph, $\delta \geq k-1$. Deduce that every k -chromatic graph has at least k vertices of degree at least $k-1$.

Or

- (b) Prove that, in a critical graph, no vertex cut is a clique.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Show that a loop less graph G contains a bipartite spanning subgraph H such that $d_H(v) \geq \frac{1}{2}d_G(v)$ for all $v \in V(G)$.

Or

- (b) Show that a graph G is bipartite if and only if it contains no odd cycles.

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