(7 pages) **Reg. No. :** ....

Code No. : 30574 E **Sub. Code : SMMA 51**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2020.

Fifth Semester

Mathematics – Core

ABSTRACT ALGEBRA – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which one of the following is not true in a vector space V

(a) $\alpha.0 = 0 \;\forall\; \alpha \;\varepsilon\; F$

(b) $0.v = 0 \;\forall\; v \;\varepsilon\; v$

(c) $\alpha.(uv)=(\alpha\, u)v$

(d) $\alpha\,(u + v) = \alpha\, u + \alpha\, v$

2. In a vector space, the set of all vectors under addition is a

(a) field (b) ring

(c) group (d) abelian group

3. If $\dim A = 4$, $\dim B = 3$ and $\dim(A + B) = 6$ then $\dim(A \cap B) = ?$

(a) 1 (b) 8

(c) 4 (d) 2

4. If A and B are any two subspaces of a vector space V then

(a) $\dim A + \dim B \leq \dim V$

(b) $\dim(A + B) \leq \dim V$

(c) $\dim A + \dim B \geq \dim V$

(d) $\dim A + \dim B = \dim V$

5. If $T : V \to W$ is a linear transformation then

(a) $\dim V \leq \dim T(V)$

(b) $\dim V = \dim T(V)$

(c) $\dim V \geq \dim T(V)$

(d) None of these

6. If $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$ and $f(t) = t - 2$ then

$\|f\| = ?$

(a) $\sqrt{\frac{7}{3}}$ (b) $\frac{3}{7}$

(c) $\frac{7}{3}$ (d) $\frac{4}{\sqrt{3}}$

7. The rank of the matrix $\begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ is

(a) 1 (b) 2

(c) 3 (d) 4

8. Choose the matrix for which the inverse exists

(a) $\begin{pmatrix} 2 & 1.5 \\ 4 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & 3 \\ 2 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} \frac{1}{10} & \frac{2}{5} \\ \frac{1}{20} & \frac{1}{5} \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

9. The characteristics equation of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ is ————

(a) $x^2 - 2x + 7 = 0$ (b) $x^2 + 2x - 5 = 0$

(c) $x^2 - 2x - 5 = 0$ (d) $x^2 - 2x + 5 = 0$

10. The quadratic form of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is ————

(a) $x^2 + y^2$ (b) $2xy$

(c) $x^2 + 2xy$ (d) $(x + y)^2$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If A and B are subspaces of a vector space V then prove that $A \cap B$ is also a subspace of V. In $A \cup B$ a subspace of V?

Or

(b) If $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(a, b) = (2a - 3b,\ a + 4b)$ then verify whether T is a linear transformation or not.

12. (a) Prove that $S = \{(2, -3, 1), (0, 1, 2), (1, 1, 2)\}$ is a basis for $V_3(\mathbb{R})$.

Or

(b) Let V be a finite dimensional vector space over a field F and A be a subspace of V. Prove that there exists a subspace B of V such that $V = A \oplus B$.

13. (a) Prove that an orthogonal set of non-zero vectors in an inner product space is linearly independent.

Or

(b) Find the linear transformation determined by the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ with respect to the standard basis $\{e_1, e_2, e_3\}$ in $V_3(\mathbb{R})$.

14. (a) Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$.

Or

(b) Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$.

15. (a) Prove that the characteristic roots of a Hermitian matrix are real.

Or

(b) Find the matrix of the bilinear form $f(x, y) = x_1 y_2 - x_2 y_1$ with respect to the standard basis in $V_2(\mathbb{R})$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that $\mathbb{R}^n$ is a vector space over $\mathbb{R}$.

Or

(b) If A and B are two subspaces of a vector space V over a field F then prove that $\frac{A + B}{A} \cong \frac{B}{A \cap B}$.

17. (a) (i) Prove that any subset of a linearly independent set in a vector space V is linearly independent.

(ii) Let V be a vector space over a field F. Let $S, T \leq V$. Prove that $L(S \cup T) = L(S) + L(T)$.

Or

(b) Let V be a finite dimensional vector space over a field F. If W is a subspace of V then show that $\dim(V / W) = \dim V - \dim W$.

18. (a) Prove that every finite dimensional inner product space has an ortho-normal basis.

Or

(b) If V and W are vector spaces of dimensions m, n respectively over F then show that $L(V, W)$ is a vector space of dimension $m.n$ over F.

19. (a) State and prove Cayley-Hamilton theorem.

Or

(b) Find the inverse of $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -2 & 0 & 0 \\ 1 & 2 & 1 & -2 \\ 0 & 3 & 2 & 1 \end{bmatrix}$ by elementary transformation.

20. (a) Find the eigen values and eigen vector of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ -4 & 4 & 2 \\ 4 & -3 & -1 \end{bmatrix}$.

Or

(b) Reduce the quadratic form

$2x_1x_2 - x_1x_3 + x_1x_4 - x_2x_3 + x_2x_4 - 2x_3x_4$ to the diagonal form using Lagrange's method.