(7 pages) **Reg. No. :** ....

Code No. : 7437 **Sub. Code : HCSM 11**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2015.

First Semester

Computer Science

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The number of rows in a truth table for a formula with n veriables is ————

   (a) n (b) $2n$

   (c) n^2 (d) 2^n

2. A statement formula which is equivalent to the given formula and expressed as sum of min terms is called ————

   (a) PCNF (b) CNF

   (c) PDNF (d) DNF

3. Let R be a relation defined on a set A, and for every $x \in A$, $(x,x) \notin R$ then R is called ————

   (a) Reflexive (b) Symmetric

   (c) Irreflexive (d) Anti symmetric

4. If $f(x) = x+2$ and $g(x) = x-2$ the $g \circ f$ is ————

   (a) $\{(x,x)/x \in R\}$ (b) $\{(x,2x)/x \in R\}$

   (c) $\{(x,2-x)/x \in R\}$ (d) $\{(x,2+x)/x \in R\}$

5. Every finite group of order n is isomorphic to permutation group of degree ————

   (a) $n+1$ (b) $2n$

   (c) n (d) n^2

6. The set of all invertible elements of a monoid from a ———— under the same operation as that of the monoid

   (a) subgroup (b) group

   (c) abelian group (d) none

7. If a closed walk in a graph contains all the edges of the graph then the walk is called ————.

   (a) Open walk (b) Euler line

   (c) Euler circuit (d) Hamiltonian circuit

8. A simple graph with 'n' vertices must be connected if it has more than ———— edges

(a) $n+1$

(b) $\frac{(n+1)(n+2)}{2}$

(c) $\frac{(n-1)(n-2)}{2}$

(d) $(n+1)(n+2)$

9. A tree in which there is exactly one vertex of degree two and all other vertices is of degree one or three is called ———— tree

(a) Binary

(b) Rooted

(c) Spanning tree

(d) Minimal spanning tree

10. A spanning tree T of a connected graph G is also called as ————

(a) Maximal tree subgraph

(b) Maximal tree of G

(c) (a) and (b)

(d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Show that $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology.

Or

(b) Define functionally complete set of connectives. Explain with suitable example.

12. (a) Let $A = \{5,6,7,8\}$ and $R = \{(x,y)/x > y\}$. Draw the graph of R and also give its matrix.

Or

(b) Let R be a relation on a set A. Then define $R = \{(a, b) \in A \times A / (b, a) \in R\}$. Prove that if (A, R) is poset then (A, R^{-1}) is also a poset.

13. (a) Show that if $(G,*)$ is a cyclic group then every subgroup of $(G,*)$ must be cyclic.

Or

(b) Find all subgroups of $(Z_6, +_6)$, where $(Z_6, +_6)$ being the group of residue classes modulo 6 under addition modulo 6.

14. (a) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-2)}{2}$.

Or

(b) Show that in a simple digraph, every node of the digraph lies in exacting one strong component.

15. (a) Define the following terms in trees :

(i) Pendant vertex

(ii) Centre of a tree

(iii) Distance between two vertices

(iv) Root of a tree.

Or

(b) Prove that every connected graph has atleast one spanning tree.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Using indirect method of proof, derive $P \to \neg S$ from the premises $P \to (Q \vee R)$, $Q \to \neg P$, $S \to \neg R$ and P.

Or

(b) Find the PCNF and PDNF of the following $S \Leftrightarrow (P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$.

17. (a) Let R be a relation on $A = \{1,2,3\}$ such that (a,b) if and only if $a+b$ is even. Find the relational matrix of $R, R^{-1}, \overline{R}$ and R^2.

Or

(b) Let $x = \{1,2,3....7\}$ and $R = \{(x,y)/x-y$ is divisible by $3\}$. show that R is an equivalence relation. Draw the graph of R.

18. (a) State and prove Lagrange's theorem.

Or

(b) Let $(G,*)$ be a finite cycle group generated by an element $G \in G$ of G is if order n, prove that $G^n = e$ and $G = \{a, a^2, a^3....a^n = e\}$ where n is the least positive integer for which $a^n = e$.

19. (a) Prove that a simple graph with n vertices and K components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

Or

(b) Show that K_n has a Hamiltonian circuit for $n \geq 3$. Obtain all the edge disjoint Hamiltonian circuits of K_7.

20\. (a) Prove that a graph is a tree, if and only if it is minimally connected.

Or

(b) If B is a circuit matrix of a connected graph G with e edges and n vertices then prove that rank of $B = e - n + 1$.
