MARKS : 75 marks

Code No:SS5837 Sub Code: PMAM22

M.Sc. (CBCS) DEGREE SPECIAL SUPPLEMENTARY EXAMINATION,

APRIL 2020

SECOND SEMESTER

MATHEMATICS

ANALYSIS II

(For those who joined in July 2017 onwards)

Time: Three hours Maximum : 75 marks

Part A – (10X1=10 marks)

Answer ALL the questions, Choose the correct answer

1. If $f \in R(\alpha)$, $m < f < M$, ϕ is continuous and $h(x) = \phi(f(x))$, then
   a) h(x) =0
   b) h is Riemann integrable
   c) h is not Riemann integrable
   d) h need not be Riemann integrable

2. If f is continuous on [a,b] then
   a) $f \in R(\alpha)$
   b) $f \notin R(\alpha)$
   c) Only real valued continuous functions on are integrable.
   d) None of the above is true

3. A continuous mapping v of an interval [a, b] into R^K is called ------------- R^K.
   a) an arc b) a closed curve
   c) a curve d) a straight line

4. If $\gamma(a) = \gamma(b)$ then γ is said to be a ----------- curve
   a) closed b) open
   c) mapping d) arc

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5. With usual notations in the exponential function, for all $z \in C$.

$E(z)E(-z) =$ -----------

a) 0

b) $E(z)$

c) 1

d) None

6. If f and α have a common point of discontinuity then

a) $f(x) = 0$

b) f is riemann integrable

c) f is not riemann integrable

d) f need not be riemann integrable

7. $E(\pi i/2) =$

a) 1

b) i

c) $-i$

d) -1

8. The exponential function E is periodic with period -----

a) πi

b) $-\pi i$

c) 2π

d) $2\pi i$

9. With usual notations, $\Gamma(1/2) =$ ------------

a) 1

b) 0

c) $\sqrt{\pi}$

d) $1/\sqrt{\pi}$

10. Log Γ is -------- on $(0, \infty)$

a) convex

b) 0

c) not defined

d) not differentiable

PART B – (5 x 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b). Each answer should not exceed 250 words.

11.(a) Prove that $\mathcal{C}(X)$ is a complete metric space

(b) State and prove the fundamental theorem of calculus.

12.(a) Show by an example that a convergent series of continuous functions may have a discontinuous sum.

Or

(b) Exhibit an example to show that an everywhere discontinuous limit functions is not Reimann integrable.

13. (a) If K is compact, if $f_n \varepsilon \mathscr{C}(K)$ for n = 1, 2, 3,.... and if $\{ f_n \}$ is point wise bounded and equicontinuous on K, then prove that $\{ f_n \}$ contains a uniformly convergent subsequence .

Or

(b) Prove that $\{ f_n \}$ is a point wise bounded sequence of complex functions on a countable set E, then $\{ f_n \}$ has a subsequence $\{ f_{n k} \}$ such that $\{ f_{n k} \}$ converges for x in E.

14.(a) Show that $e^{x+y} = e^x . e^y$ and $\lim_{x \to \infty} x^n . e^{-x} = 0$

Or

(b) Suppose Σc_n converges. Put $f(x) = \Sigma c_n x^n$, $-1 < x < 1$. Prove that

$\lim_{x \to 1} f(x) = \Sigma c_n$.

15.(a) State and prove the localization theorem .

Or

(b) State and prove St' irlings's formula

PART C - (5 x 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b). Each answer should not exceed 600 words.

16. (a) Prove that $f \varepsilon \mathscr{R}(\alpha)$ on [a, b] if and only if for every $\varepsilon > 0$, there exists a partition P such that $U(p, f, \alpha) - L(p, f, \alpha) < \varepsilon$

Or

(b) Prove that f is monotonic on [a, b] and if α is continuous on [a, b] then $f \varepsilon \mathscr{R}(\alpha)$

17. (a) Prove that if $\{f_n\}$ is a sequence of continuous functions on E and if $f_n \to f$ uniformly on E then f is continuous on E.

Or

(b) Given $\{f_n\}$ is a sequence of functions defined on E and suppose $|f_n(x)| \leq M_n$ ($x \varepsilon E$, n=1, 2, 3, ...) . Prove Σf_n converges uniformly on E if ΣM_n converges.

18. (a) Prove that there exists a real continuous function on the real line which is no where differentiable.

Or

(b)) If K is compact metric space, $f_n \varepsilon \mathscr{C}(K)$ for n = 1, 2, 3,.... and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous .

19. (a) State and prove the Stone Weierstrass theorem.

Or

(b) Discuss Point wise convergence of Fourier series.

20. (a) Show that every non-constant polynomials with complex coefficients has a complex root.

Or

(b) Show that if f(x) is a periodic function with period 2π defined in $[-\pi, \pi]$, then f(x) can be expanded in a Fourier Series.