

(7 Pages)

Reg. No. : .....

Code No. : 5681

Sub. Code : ZPHM 21

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Second Semester

Physics — Core

MATHEMATICAL PHYSICS — II

(For those who joined in July 2021 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If  $A$  satisfies the condition  $AA^* = 1$ , then  $A$  will be

- (a) Hermitian                      (b) Symmetric  
(c) Unitary                        (d) Anti symmetric

2.  $A = \begin{bmatrix} 3i & 4-5i \\ -4-5i & 0 \end{bmatrix}$  is

- (a) Hermitian                      (b) Skew Hermitian  
(c) Symmetric                    (d) Skew symmetric

3. The Cauchy Riemann equation do not ensure that the function is independent of the path along which  $\Delta Z \rightarrow 0$  ———

- (a) True                              (b) False  
(c) Partially                        (d) None of these

4. The residue of  $\cot z$  at  $z = 0$  will be

- (a)  $e$                                   (b)  $e^{-1}$   
(c) 1                                    (d) 0

5. The value of  $H_1(x)$  is

- (a)  $x^2 - 1$                           (b)  $x^2 + 1$   
(c)  $x - 1$                             (d)  $2x$

6. The value of  $J_{-1/2}$  is

- (a)  $\sqrt{\frac{2}{\pi x}} \cos x$                       (b)  $\sqrt{\frac{2}{\pi x}} \sin x$   
(c)  $\sqrt{\frac{2}{\pi}} \cos x$                         (d)  $\sqrt{\frac{2}{\pi}} \sin x$

7. Periodic function of half-wave symmetry is necessarily

- (a) An even function  
(b) An odd function  
(c) Neither odd nor even  
(d) Both odd and even

Page 2

Code No. : 5681



8. The trigonometric Fourier series of a periodic function can have only \_\_\_\_\_

- (a) Cosine term
- (b) Sine term
- (c) Cosine and sine term
- (d) None of the above

9. A cycle group is always

- (a) Monoid
- (b) Abelian group
- (c) Semi group
- (d) Sub group

10.  $(1, i, -i, -1)$  is

- (a) Semi group
- (b) Sub group
- (c) Abelian group
- (d) Cyclic group

PART B —  $(5 \times 5 = 25 \text{ marks})$

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the rank of matrix A

Where  $A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ .

Or

Page 3

Code No. : 5681

(b) Solve by Carmer's rule the system of equations

$$x + 2y + 3z = 10$$

$$2x - 3y + z = 1$$

$$3x + y - 2z = 9.$$

12. (a) State and prove Cuchy's integral theorem.

Or

(b) Find the most general analytic function  $f(z) = u + iv$  for which  $v = xy$ .

13. (a) Prove that  $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$ .

Or

(b) Obtain the generating function of Bessel function.

14. (a) State and prove convolution theorem in Fourier transform.

Or

(b) Obtain Fourier series for the expansion  $f(x) = x \sin x$  in the interval  $-\pi < x < \pi$ .

Page 4

Code No. : 5681

[P.T.O]



15. (a) Define isomorphic and homomorphic groups. Differentiate between them and given at least two properties of each.

Or

- (b) Prove that the two dimensional representation of matrices  $C_4$ .  
i.e.

$$T(E) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, T(A) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$T(A^2) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \text{ and } T(A^3) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ is reducible. Hence express it in terms of one dimensional representation.}$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the eigen values and vectors of

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}.$$

Or

- (b) Find the characteristic equation of the

$$\text{matrix } A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix} \text{ and Hence find}$$

$$A^{-1}.$$

Page 5

Code No. : 5681

17. (a) Apply Calculus of residues to show that

$$\int_0^{\infty} \frac{dx}{(x^2+1)(x^2+9)} = \frac{\pi}{24}.$$

Or

- (b) State and prove Laurent's theorem of a complex function.

18. (a) Obtain the series solution of Hermite differential equation  $\frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2ny = 0$ .

Or

- (b) What is Spherical Bessel function? Show that  $\int_{-\infty}^{\infty} j_n(x) j_m(x) dx = \frac{\pi}{2n+1} \delta_{mn}$ .

19. (a) Find the Laplace transform of

(i)  $\frac{\sin at}{t}$

(ii)  $\frac{\sin t}{t}$

(iii) Does the transform of  $\frac{\cos at}{t}$  exist?

Or

Page 6

Code No. : 5681



- (b) Define Laplace transform. Explain the following
- (i) Linearly property
  - (ii) Second translation property.
20. (a) Explain the concept of group representation prove the great orthogonality theorem in group theory.

Or

- (b) State and prove Schur's Lemma-I.

