(6 pages) **Reg. No. :** ....

Code No. : 30946 E **Sub. Code : FECA 11**

B.C.A. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2024.

First Semester

Computer Application

Elective — DISCRETE MATHEMATICS — I

(For those who joined in July 2024 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let $R = \{(1, 3), (1, 7), (2, 3), (2, 7)\}$. Then $\text{Dom}(R)$ = ____________.

   (a) {1, 1, 2, 2} (b) {3, 7, 3, 7}

   (c) {1, 2} (d) {3, 7}

2. A path that begins and ends at the same vertex is called a ____________.

   (a) graph (b) edge

   (c) relation (d) cycle

3. The set of all images of elements of A is called ____________.

   (a) relation (b) function

   (c) domain (d) range

4. If every element of A is assigned to the same element of B, then the function said to be a ____________.

   (a) identify function (b) one to one function

   (c) constant function (d) onto function

5. $5+5=10 \vee 1>2$. Truth value of this statement is ____________.

   (a) false (b) true

   (c) true or false (d) true and false

6. The contrapositive of $p \to q$ is the proposition ____________.

   (a) $q \to p$ (b) $\sim p \to \sim q$

   (c) $\sim q \to p$ (d) $\sim q \to \sim p$

7. If A is a square matrix of order n in which every non-diagonal element is zero and every diagonal element is 1, then the matrix A is called a ____________ matrix.

   (a) zero (b) diagonal

   (c) unit (d) scalar

8. A square matrix A is called an involutory matrix if $A^2 =$ ____________.

(a) $A^* A$ (b) $A^2 A$

(c) A^2 (d) I

9. A path with no repeated vertex is called as ____________.

(a) path (b) self loop

(c) simple path (d) trail

10. A graph with n vertices is ____________ if either r or n or both are even.

(a) regular (b) r-regular

(c) cycle (d) r-cycle

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) (i) Define composition of relation.

(ii) Find the composition of the Relations $R_1 = \{(1, 2), (1, 6), (2, 4), (3, 4), (3, 6), (3, 8)\}$ and $R_2 = \{(2, x), (4, y), (4, z), (6, z), (8, x)\}$.

Or

(b) $A = \{a, b, c\}$ and $M_R = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. Find the relation R defined on A.

12. (a) Let $f : R \to R$ be defined by $f(x) = x + 1$ and Let $g : R \to R$ be defined as $g(x) = 2x^2 + 3$. Find $f \circ g$ and $g \circ f$. Is $f \circ g = g \circ f$?

Or

(b) The composition of any function with the identify function is the function itself. Prove it.

13. (a) Verity that the proposition $p \vee \sim (p \wedge q)$ is a tautology.

Or

(b) Show that

$$p \to (q \to r) \Leftrightarrow p \to (\sim q \vee r) \Leftrightarrow (\sim p \wedge q) \vee r.$$

14. (a) Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{pmatrix}$.

Or

(b) Paraphrase addition of matrices and subtraction of matrices with example.

15. (a) Define bipartite graph. Show that the graph C_6 is bipartite.

Or

(b) Prove that the number of spanning subgraphs of a graph G with m vertices is 2^m.

[P.T.O.]

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let $A = \{1, 2, 3\}$. Check whether the following relations are reflexive, symmetric, anti symmetric or transitive.

(i) $R = \{(1, 1), (2, 2), (3, 3), (1, 3), (1, 2)\}$

(ii) $R = \{(1, 1), (2, 2), (1, 3), (3, 1)\}$

(iii) $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$

Or

(b) Illustrate closure operations on relations with example.

17. (a) Examine one to one function and onto function with example.

Or

(b) If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are bijections then $(g \circ f)^{-1} = f^{1} \circ g^{-1}$. Prove it.

18. (a) Show that $p \wedge (q \vee r)$ is equivalent to $(p \wedge q) \vee (p \wedge r)$.

Or

(b) Determine the contrapositive, the converse and the inverse of the conditional statement "The Team A wins whenever it is raining".

19. (a) Show that $\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$ is orthogonal. Determine the value of $|A|$.

Or

(b) Discuss the following with example :

(i) symmetric matrix and skew-symmetric matrices.

(ii) complex matrix

(iii) conjugate matrix.

20. (a) Let G be a simple graph with 12 edges. If G has 6 vertices of degree 3 and the rest of the vertices have degree less than 3, then find the

(i) minimum number of vertices and

(ii) maximum number of vertices.

Or

(b) State and prove the handshaking theorem.