

(7 pages)

Reg. No. :

Code No. : 9072

Sub. Code : PMAM 14

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2017.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2017 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $y_1(x)$ and $y_2(x)$ are two solutions
 $y'' + P(x)y' + Q(x)y = 0$, then $W =$

- (a) ce^{-SPdx}
(b) ce^{-SQdx}
(c) ce^{+SPdx}
(d) ce^{+SQdx}

2. Two linearly independent solutions of $y'' - y = 0$ are

- (a) 1 and e^x (b) 1 and e^{-x}
(c) e^x and e^{-x} (d) None

3. The equation $y' = y$ has a power series solution with radius of convergence R where

- (a) $R > 0$ (b) $R < 0$
(c) $R = 0$ (d) None

4. The particular solution of
 $(1-x^2)y'' - 2xy' + p(p+1)y = 0$ where p is a constant is known as

- (a) Bessels equation (b) Legendre equation
(c) Hermite equations (d) AIRY's equation

5. The singular point of $x^2y'' + 2xy' - 2y = 0$ is

- (a) 1 (b) ∞
(c) 0 (d) -1

6. The Rodriques formula $p_n(x)$ is

- (a) $\frac{1}{2^n n!} \frac{d^n}{dx^n} [x^2 - 1]^n$ (b) $\frac{1}{2^n n!} \frac{d^n}{dx^n} [x^2 + 1]^n$
(c) $\frac{1}{2^n n!} \frac{d^n}{dx^n} [x^2 - 1]^n$ (d) $\frac{1}{2^n n!} \frac{d^n}{dx^n} [x^2 + 1]^n$

Page 2

Code No. : 9072



7. The value of $J_{y_2}(x)$ is

- (a) $\sqrt{\frac{2}{\pi x}} \sin x$ (b) $\sqrt{\frac{2}{\pi x}} \cos x$
(c) $\sqrt{\frac{2}{\pi x}} \tan x$ (d) None

8. The famous $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ whose sum was discovered by

- (a) Newton (b) Euler
(c) Bessel (d) Legendre

9. The vanishing or non vanishing of the Wronskian $w(t)$ of two solutions _____ on the circle of t .

- (a) Depends (b) Does not depend
(c) Either (a) or (b) (d) None

10. By solving the linear systems using auxillary equation. If m_1 and m_2 all distinct real numbers then the roots are _____.

- (a) Same (b) Distinct
(c) Trivial (d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, by choosing either (a) or (b).

11. (a) If $y_1(x)$ and $y_2(x)$ are any two solutions of the equation $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$. Then prove that $W = W(Y_1, Y_2)$ is either identically zero or never zero on $[a, b]$.

Or

- (b) The equation $y'' - y = 0$ has a solution $y_1 = e^x$. Find y_2 and the general solution.

12. (a) Find a power series solution of the equation $y' = y$.

Or

- (b) Find the general solution of $(1+x^2)y'' + 2xy' - 2y = 0$ in terms of power series.

13. (a) Find the indicial equation and its roots the equation $x^3y'' + (\cos 2x - 1)y' + 2xy = 0$.

Or

- (b) Find the first three terms of the Legendre series $f(x) = e^x$.



14. (a) Prove that : $\frac{d}{dx}[J_0(x)] = -J_1(x)$.

Or

- (b) Show that : $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

15. (a) Prove that if the solutions of the homogeneous system are linearly independent on $[a, b]$. Then the system is the general solution of homogeneous solution of this interval.

Or

- (b) Find the general solution of $\frac{dx}{dt} = 2x$,
 $\frac{dy}{dt} = 3y$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, by choosing either (a) or (b).

16. (a) Show that $y = c_1 e^x + c_2 e^{2x}$ is the general solution of $y'' - 3y' + 2y = 0$ on any interval and find the particular solution for which $y(0) = 1$ and $y'(0) = 1$.

Or

Page 5

Code No. : 9072

- (b) If y_1 is a non zero solution of the equation $y'' + P(x)y' + Q(x)y = 0$ and $y_2 = v y_1$, where v

is given by the formula $v = \int \frac{1}{y_1^2} e^{-\int P dx} dx$ is the

second solution. Show by computing the Wronkian that y_1 and y_2 are linearly independent.

17. (a) Show that $y = (1+x)^p$ is a power series solution of the equation $(1+x)y' = p y$, $y(0) = 1$.

Or

- (b) Find the general solution of $(1-x^2)y'' - 2xy' + p(p+1)y = 0$, where p is a constant.

18. (a) Show that the indicial equation of the equation $x^2 y'' + xy' + x^2 y = 0$ has only one root and prove that $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}$.

Or

- (b) Show that :

$$\int_{-1}^1 p_m(x) p_n(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2}{2n+1} & \text{if } m = n \end{cases}$$

Page 6

Code No. : 9072



19. (a) Prove that $J_{-m}(x) = (-1)^m J_m(x)$.

Or

- (b) Prove that

$$\int_0^1 x J_p(\lambda_m x) J_p(\lambda_n x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{1}{2} J_{p+1}(\lambda_n)^2 & \text{if } m = n \end{cases}$$

20. (a) Show that the Wronskian of the two solutions in distinct complex roots is given by $w(t) = (A_1 B_2 - A_2 B_1) e^{2at}$ and prove that $A_1 B_2 - A_2 B_1 \neq 0$.

Or

- (b) Find the general solution of

$$\frac{dx}{dt} = 3x - 4y, \frac{dy}{dt} = x - y.$$

