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M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2012.

Fourth Semester

Mathematics

**CALCULUS OF VARIATIONS AND INTEGRAL
EQUATIONS**

(For those who joined in July 2008 and afterwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

1. Define Lagrange multiplier.
2. State the condition satisfied at a stationary point of the function $z = f(x, y)$.
3. What is a transversality condition?
4. What is a characteristic value problem?
5. Define an integral equation.

6. Define a Fredholm integral equation.
7. Define an influence function.
8. State the divergence theorem.
9. State the condition for the existence of continuous solution to the Fredholm equation of the first kind.
10. Give an example of a real symmetric Kernel.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, Choosing either (a) or (b).

11. (a) If $I(y) = \int_0^1 \sqrt{1+y'^2} dx$; calculate $I(x)$ and $I(\cosh x)$.

Or

- (b) Find the function $y = f(x)$ which minimizes the integral $I = \int_0^1 (1+y'^2) dx$ satisfying the end condition $y(0) = 0; y(1) = 1$.

12. (a) Use the calculus of variation to find the shortest distance between the line $y = x$ and the parabola $y^2 = x - 1$.

Or

- (b) Determine the equation of the shortest arc in the first quadrant which passes through the point $(0,0)$ and $(1,0)$ and encloses a prescribed area A with the X-axis where $A \leq \frac{\pi}{8}$.

13. (a) Verify that $U(x)=x$ and $V(x)=x^2$ are linearly independent solutions of the equation $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$.

Or

- (b) Determine $p(x)$ and $q(x)$ such that the equation $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$ can be expressed in the form $\frac{d}{dx} \left(p \frac{dy}{dx} \right) + qy = 0$.

14. (a) Obtain an approximate solution of the integral equation $y(x) = \int_0^1 \sin(x\xi) y(\xi) d\xi + x^2$ by replacing $\sin(x\xi)$ by the first term of its power series expansion.

Or

- (b) Determine the most general form of the function $F(x)$ for which the integral equation $y(x) = \int_0^{2\pi} \sin(x+\xi) y(\xi) d\xi = F(x)$ of first kind, possesses a solution.

15. (a) Apply iterative method and obtain the results of two successive substitutions to the equation $y(x) = \int_0^x (x+\xi) y(\xi) d\xi + 1$ taking $y^0(x) = 1$.

Or

- (b) Show that the iterative procedure will converge when $|\lambda| < 3$ for the equation $y(x) = \lambda \int_0^1 x\xi y(\xi) d\xi + 1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State the brachistochrone problem and prove that the extremals of the brachistochrone problem are cycloids.

Or

- (b) Derive the Euler equation of the problem $\int_{x_1}^{x_2} [a(x)y''^2 - b(x)y'^2 + c(x)y^2] dx = 0$; obtain the associated natural boundary condition.

- (a) Define the term generalized coordinates, generalized velocity and generalized force. Derive the Lagrange's equation for the compound pendulum.

Or

- (b) Find the extremals of the variable problem $\int_0^5 y''^2 dx = 0$ subject to the conditions $\int_0^\pi y^2 dx = 1$; $y(0) = y''(0) = 0$.

3. (a) Obtain the Green's function for the Bessel operator of order n , $Ly = \frac{d}{dx} \left(x \frac{dy}{dx} \right) - \frac{n^2}{x} y$ relevant to the end condition $y(0) = y(1) = 0$.

Or

- (b) Transform the problem $\frac{d^4 y}{dx^4} + \Phi(x) = 0$; $y(0) = y'(0) = y(1) = y'(1) = 0$.

19. (a) Give a detailed account of cause and effect and influence function with reference in integral equation with suitable examples.

Or

- (b) Explain the method of solving Fredholm equation with separable kernels.

20. (a) Prove that the exact solution of the integral equation $y(x) = \lambda \int_0^1 x\xi y(\xi) d\xi + 1$ is given by

$$y(x) = 1 + \frac{3\lambda x}{2(3-\lambda)}; \lambda \neq 3.$$

Or

- (b) Obtain the exact solution of the equation $y(x) = \sin x + \lambda \int_0^{2\pi} \cos(x+\xi) y(\xi) d\xi$.