(7 pages) **Reg. No. :** ....

Code No. : 6363 **Sub. Code : HMAM 31**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2014.

Third Semester

Mathematics

COMPLEX ANALYSIS

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer the following questions.

Choose the correct answer :

1. $\frac{1}{R} = \lim\sup_{n\to\infty} \sqrt[n]{|a_n|}$ is known as ———— formula for the radius of convergence.

   (a) Cauchy's

   (b) Cauchy-Riemann's

   (c) Hadamard's

   (d) None of these

2. If $f(z) = u + iv = z^2$ then $\frac{\partial u}{\partial y} =$ ————.

   (a) $2x$ (b) $2y$

   (c) $-2y$ (d) $-2x$

3. The function arc $\cos z$ is a ———— function.

   (a) single-valued (b) multiple-valued

   (c) analytic (d) none of these

4. $w = s(z) = \frac{az+b}{cz+d}$ is said to be normalized if $ad - bc =$ ————.

   (a) 0 (b) 1

   (c) 2 (d) –1

5. With usual notation, $\int_{\gamma 1 - \gamma 2} f dz =$ ————.

   (a) $\int_{\gamma 1} f dz + \int_{\gamma 2} f dz$ (b) $\int_{\gamma 1} f dz - \int_{\gamma 2} f dz$

   (c) $\int_{-\gamma 1} f dz + \int_{\gamma 2} f dz$ (d) None of these

6. If C is a circle with center 'a' and radius 'ρ', then $\int_C |dz| =$ ____________.

(a) 0 (b) 2ρ

(c) 2π (d) $2\pi\rho$

7. If $f(z)$ is analytic in a region $0 < |z-a| < \delta$, then 'a' is called ____________.

(a) a singularity

(b) a zero element

(c) an isolated singularity

(d) none of these

8. If γ lies inside of a circle, then $n(\gamma, a) =$ ____________ for all points 'a' outside of the same circle.

(a) 1 (b) 2

(c) 0 (d) 3

9. The residue of $f(z) = \dfrac{e^z}{(z-a)(z-b)}$ at $z = a$, $a \neq b$ is ____________.

(a) $\dfrac{e^a}{(a-b)}$ (b) $\dfrac{e^a}{(b-a)}$

(c) e^a (d) None of these

10. The poles of $\dfrac{1}{z^2+5z+6}$ are ____________.

(a) $-2, -3$ (b) $1, -1$

(c) $-2, -1$ (d) $-3, 1$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Expand $(1-z)^{-m}$, m a positive integer, in powers of z.

Or

(b) Expand $\dfrac{2z+3}{z+1}$ in powers of $z-1$. What is the radius of convergence?

12. (a) If z_1, z_2, z_3, z_4 are distinct points in the extended plane and T any linear transformation, then prove that $(Tz_1, Tz_2, Tz_3, Tz_4) = (z_1, z_2, z_3, z_4)$.

Or

(b) Find the linear transformation which carries $0, i-i$ into $1, -1, 0$.

13. (a) Find the fixed points of the linear transformation $w = \dfrac{3z-4}{z-1}$. Is the transformation elliptic, hyperbolic, or parabolic?

Or

(b) Compute $\int_{|z|=1} |z-1|\,|dz|$.

14. (a) State and prove Liouville's theorem.

Or

(b) State and prove Morera's theorem.

15. (a) Compute $\int_0^{\pi} \dfrac{d\theta}{a+\cos\theta}, a>1$.

Or

(b) Compute $\int_0^{\pi} \log\sin\theta \, d\theta$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Abel's theorem.

Or

(b) Prove that every rational function has a representation by partial fractions.

17. (a) If the consecutive vertices z_1, z_2, z_3, z_4 of a quadrilateral lie on a circle, prove that $|z_1 - z_3| \cdot |z_2 - z_4| = |z_1 - z_2| \cdot |z_3 - z_4| + |z_2 - z_3| \cdot |z_1 - z_4|$.

Or

(b) Find the linear transformation which carries the circle $|z| = 2$ into $|z+1| = 1$, the point -2 into the origin and the origin into $\dot{L}$.

18. (a) State and prove Cauchy's theorem for a rectangle.

Or

(b) Prove that the line integral $\int_{\gamma} p\,dx + q\,dy$, defined in Ω, depends only on the ends points of γ if and only if there exists a function $U(x, y)$ in Ω with partial derivatives $\dfrac{\partial U}{\partial x} = p, \dfrac{\partial U}{\partial y} = q$.

19. (a) State and prove Cauchy's integral formula, by proving the required results.

Or

(b) State and prove Taylor's theorem by proving the result on extending an analytic function at removable singularities.

20. (a) State and prove analytically the maximum principle.

Or

(b) State and prove argument principle. Also state Rouche's theorem and deduce the proof from argument principle.

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