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Code No. : 7857   Sub. Code : PPHM 12

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2019.

First Semester

Physics

Core – MATHEMATICAL PHYSICS – I

(For those who joined in July 2017 onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

Choose the correct answer :

1. The curl of a vector field can be represented as
   _____

   (a) Curl A   (b) $\nabla \times A$

   (c) $\nabla . A$ (d) $\nabla^2 A$

2. A field is irrotational if _____

   (a) $grad A = 0$ (b) $div A = 0$

   (c) $curl A = 0$ (d) $\nabla^2 A = 0$

3. The eigen values of the matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
   are _____

   (a) $e^{\pm i\theta}$ (b) $e^{\pm 2i\theta}$

   (c) $e^{\pm 3i\theta}$ (d) $e^{\pm i\theta/2}$

4. Which one of the following matrices is Hermitian?

   (a) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ (b) $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

   (c) $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ (d) $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$

5. The value of $J_{-1/2}\left(\dfrac{\pi}{2}\right)$ is

   (a) 0   (b) 1

   (c) $\dfrac{\pi}{2}$ (d) $\dfrac{2}{\pi}$

6. The value of $L_1(x)$ is

   (a) 1   (b) $1 - x$

   (c) $1 + x$ (d) 0

7. Fourier transform of which of the following functions does not exit?

(a) $e^{-|x|}$

(b) xe^{-x^2}

(c) e^{x^2}

(d) e^{-x^2}

8. The average of the function $f(x) = \sin x$ in the interval $(0, \pi)$ is —————

(a) $\dfrac{1}{2}$

(b) $\dfrac{2}{\pi}$

(c) $\dfrac{1}{\pi}$

(d) $\dfrac{4}{\pi}$

9. Laplace transform of $J_0(t)$ is

(a) $\dfrac{1}{S}$

(b) $\dfrac{1}{\sqrt{S^2 + 1}}$

(c) $\dfrac{1}{S^2 + 1}$

(d) $\dfrac{1}{\sqrt{1 - S^2}}$

10. Laplace transform of $\cos at$ is

(a) $\dfrac{s}{s^2 + a^2}$

(b) $\dfrac{a}{s^2 + a^2}$

(c) $\dfrac{a}{s^2 - a^2}$

(d) $\dfrac{s}{s^2 - a^2}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find the value of a, b and c such that

$$F = (3x - 4y + 9z)i + (cx + 5y - 2z)j + (x - by + 7z)k \text{ is irrotational.}$$

Or

(b) Using Green's theorem evaluate $\int_C (x^2 y\, dx + x^2\, dy)$ where C is boundary described counter-clockwise of the triangle with vertices $(0, 0), (1, 0), (1, 1)$.

12. (a) Show that $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$ where σ_x, σ_y and σ_z are Pauli spin marices.

Or

(b) Show that the eigen values of a Hermitian matrix are real.

13. (a) Prove that $x J_x'(x) = -n J_n(x) + x J_{n-1}(x)$.

Or

(b) Prove that $(n+1) L_{n+1}(x) = (2n + 1 - x) L_n(x) - n L_{n-1}(x)$.

14. (a) State and prove modulation theorem of Fourier transform.

Or

(b) Find the sine transform of $\dfrac{e^{-ax}}{x}$.

15. (a) Find Laplace transforms of

(i) $\cosh(at)$

(ii) $\sin at$.

Or

(b) Find the inverse Laplace transform of $\dfrac{e^{-\pi s}}{s^2+1}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) State and prove Gauss divergence theorem.

Or

(b) Show that $r^n \vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n=-3$ $\vec{r}$ is position vector of a point.

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17. (a) State and prove Cayley Hamilton theorem.

Or

(b) Find the eigen values any eigen vectors of the following matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

18. (a) Obtain the series solution of Languve differential equation.

Or

(b) Obtain the series solution of Bessel differential equation.

19. (a) Find the Fourier transform of e^{-r^2/a^2} where a is a constant and $r=\sqrt{x^2+y^2+z^2}$.

Or

(b) Find the cosine transform of a function of x which is unity for $0<x<a$ and zero for $x \ge a$ what is the function whose cosine transform is $\sqrt{\dfrac{2}{\pi}}\dfrac{\sin ap}{p}$.

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20. (a) State and prove extension theorem is Laplace transform.

Or

(b) Obtain Laplace transform of the function $f(t) = \sinh at \sin at$.

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