(7 pages) **Reg. No. :** .....

Code No. : 6781 **Sub. Code : KPHM 21**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2017.

Second Semester

Physics

MATHEMATICAL PHYSICS — II

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Cauchy Residue theorem tells $\int_C f(z)\,dz$

(a) $= 2\pi i \sum_{i=1}^{n} \underset{z=z_k}{\text{Res}} f(z)$ (b) $= 4\pi\, i \sum_{i=1}^{n} \underset{z=z_k}{\text{Res}} f(z)$

(c) $= -4\pi i \sum_{i=1}^{n} \underset{z=z_k}{\text{Res}} f(z)$ (d) $= 4\pi \sum_{i=1}^{n} \underset{z=z_k}{\text{Res}} f(z)$

2. The principal part of the Laurent series consists of finite terms, the singularity of a function $f(z)$ at $z = z_0$ is called

(a) Pole

(b) Isolated essential singularity

(c) Removable singularity

(d) None of the above

3. The group of order lesser than 4 is

(a) always cyclic

(b) always non-cyclic

(c) may or may not be cyclic

(d) cyclic under some conditions

4. The group formed by three cube roots of unity is

(a) non-Abelian

(b) Abelian but non-cyclic

(c) Abelian and cyclic

(d) None of the above

5. Let $P_n(x)$ be Legendre Polynomial, then $P_n'(-x)$ is equal to

(a) $(-1)^{n+1} P_n'(x)$ (b) $(-1)^n P_n'(x)$

(c) $(-1)^n P_n(x)$ (d) $P_n''(x)$

6. Which of the following is not correct?

(a) $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$

(b) $H_{2n+1}(0) = 0$

(c) $H'_{2n}(0) = 0$

(d) $H'_{2n+1}(0) = 0$

7. The order of the partial differential equation $\frac{\partial u}{\partial t} - 5\frac{\partial^2 u}{\partial x^2} = 0$ is

(a) 1 (b) 2

(c) 3 (d) None of these

8. Find out which one of the following partial differential equation is non-linear

(a) $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ (b) $x^2\frac{\partial^3 R}{\partial y^3} = y^3\frac{\partial^2 R}{\partial x^2}$

(c) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 1$ (d) None of these

9. What is the rank of tensor x^{rsq}_{rq} ?

(a) 5 (b) 3

(c) 2 (d) 1

10. In which coordinate system does the intrinsic derivative of convariant vector coincide with the total derivative

(a) Cartesian (b) Cylindrical

(c) Spherical (d) None of these

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Write all possible Laurent series for the function $f(z) = \frac{1}{z(z+2)^3}$ about the pole $Z = -2$, using appropriate Laurent series.

Or

(b) Find the residue of $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z = 1$.

12. (a) Write any five main features of reducible and irreducible representation.

Or

(b) State and prove lemma for the irreducible representation of a group.

13. (a) Deduce Rodrigue's formula for Hermit polynomial.

Or

(b) Prove that

$$\int_{-1}^{1} x^2 P_{n+1}(x) \cdot P_{n-1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

14. (a) Derive vibrating string equation.

Or

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} - 2u$, $u(x, 0) = 10e^{-x} - 6e^{-4x}$ the boundary value problem by using Laplace transform.

15. (a) Prove that the direct product of two tensors results in a new tensor of rank equal to the sum of rank of these tensors.

Or

(b) What do you mean by symmetric and antisymmetric tensors? Show that any contravarient or covariant tensor of second rank can be expressed as the sum of a symmetric and antisymmetric tensor of same rank.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Show that the real and imaginary parts of the function $w = \log z$ satisfy the Cauchy-Riemann equations when z is not zero.

Or

(b) State and deduce Taylor's series theorem.

17. (a) State and prove great orthogonality theorem.

Or

(b) State and prove Schur's lemma for the irreducible representation of a group.

18. (a) Find Rodrigue's formula for Legendre Polynomial.

Or

(b) Derive an expression for the generating function for the Legendre polynomials.

19. (a) Solve $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$, $0 < x < 3$, $t > 0$, given that

$u(0,t) = u(3, t) = 0$, $u(x,0) =$

$$5\sin 4\pi x - 3\sin 8\pi x + 2\sin 10\pi x,$$

$|u(x, t)| < M$ where the last condition states that u is bounded for $0 < x < 3$, $t > 0$.

Or

(b) Find the displacement $w(x,t)$ of an elastic string subject to the following conditions,

(i) The string is initially at rest on the x-axis from $x = 0$ to ∞ ("semi-infinite string")

(ii) For $t > 0$ the left end of the string is moved in a given fashion, namely

$$w(0, t) = f(t) = \begin{cases} \sin t & \text{if} \quad 0 \le t \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

(iii) Further more $\lim_{x \to \infty} w(x, t) = 0$ for $t \ge 0$.

20. (a) Obtain tensor form of gradient, divergence, Laplacian and curl.

Or

(b) Discuss the application of tensor analysis to the dynamics of a particle.
