(6 pages) **Reg. No. :** ....

Code No. : 5831 N **Sub. Code : PMAM 11**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2020.

First Semester

Mathematics – Core

ALGEBRA – I

(For those who joined in July 2020 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The kernel of a homomorphism $f: G \to G'$ is

   (a) a normal subgroup of G

   (b) {e}

   (c) a normal subgroup of G'

   (d) a subgroup of G'

2. If G is a non-abelian subgroup of order 6, then

(a) $G \cong S_2$ (b) $G \cong S_3$

(c) $G \cong S_6$ (d) None of the above

3. The smallest non-abelian group is

(a) S_2 (b) S_3

(c) Z (d) N

4. If G is a group having 99 elements and H is a subgroup with 11 elements, then $i(H) =$

(a) 9! (b) 11

(c) 9 (d) 11!

5. Which of the following is an even permutation

(a) (1, 2, 3)(1, 2)

(b) (2, 3)(3, 4, 5)

(c) (1, 2, 3, 4, 5, 6)

(d) (1, 2)(1, 3)(1, 4)(2, 5)

6. If Z is the center of a group G then $a \in Z$ if and only if N(a) =

(a) Z (b) G

(c) $\{a\}$ (d) $\{e\}$

7. Let G be group or order 72. G has ___________ number of 3-Sylow subgroups

(a) 1 (b) 4

(c) either 1 or 4 (d) 0

8. 2- Sylow subgroup of S_4 is

(a) {(1, 2),(3,4),e}

(b) {(1, 2),(4,3),e}

(c) {(1, 2),(3,4),(1, 2,3,4), e}

(d) {(1, 2),(3,4),(1, 2),(3,4),e}

9. The number of non-isomorphic abelian groups of order 2^4 is

(a) 4 (b) 5

(c) 7 (d) 1

10. Let G be a group and let $T = G \times G; D = \{(g, g) : g \in G\}$. Then D is normal in T if and only if

(a) T is abelian

(b) D is abelian

(c) G is abelian

(d) G is non-abelian

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) If H and K are subgroups of a group G, then prove that HK is a subgroup of G if and only if HK = KH.

Or

(b) Define the terms homomorphism, Ker ϕ. Prove that if $\phi : G \rightarrow \overline{G}$ is a homomorphism, then $\phi(e) = \bar{e}; (\phi(x))^{-1} = \phi(x^{-1})$.

12. (a) Define Automorphism. If G is a group, then A(G)- the set of all automorphisms of G is also a group.

Or

(b) Define inner automorphism. Prove that $I(G) \cong \frac{G}{Z}$, where $I(G)$ is the group of inner automorphisms of G and Z is the centre of G.

13. (a) Prove that every permutation is a product of its cycles.

Or

(b) Define normalizer of an element in a group. Prove that if G is a finite group, then the number of elements conjugate to a in G is the index of the normalizer of a in G.

14. (a) Define $n(k)$. Prove that

$$n(k) = 1 + p + p^2 + .. + p^{(k-1)}.$$

Or

(b) Prove that any group G of order $11^2 \times 13^2$ is abelian.

15. (a) Suppose that G is the internal direct product of $N_1, N_2, ... N_m$. Then, for $i \neq j$, $N_i \cap N_j = (e)$ and if $a \in N_i, b \in N_j$; then $ab = ba$.

Or

(b) If G and G' are isomorphic abelian groups, then $G(s)$ and $G'(s)$ are isomorphic.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$ respectively, then prove that $o(HK) = \dfrac{o(H)o(K)}{o(H \cap K)}$

Or

(b) State and prove Cauchy's theorem for abelian groups.

17. (a) State and prove Cayley's theorem.

Or

(b) Define a simple group. If G is a finite group and $H \neq G$ is a sub group of G such that $o(G) + i(H)!$, then prove that G can not be simple.

18. (a) Define and derive class equation of a finite group G.

Or

(b) Let p be prime number. If $o(G) = p^n$, then prove that $Z(G) \neq \{e\}$. Deduce that if $o(G) = p^2$, then G is abelian.

19. (a) State Sylow's theorem and give the third proof.

Or

(b) State and prove the third part of Sylow's theorem.

20. (a) Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

Or

(b) Prove that the internal direct product of groups is isomorphic to their external direct product.
