Reg. No. : ....

Code No. : 5835 **Sub. Code : PM AM 15**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2020.

First Semester

Mathematics — Core

NUMERICAL ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. If $f(x) = \frac{1}{x}$, then the divided difference of $f(a, b, c)$ is ______

   (a) $-\frac{1}{ab}$ (b) $\frac{1}{ab}$

   (c) $-\frac{1}{abc}$ (d) $-\left(\frac{a+b}{a^2b^2}\right)$

2. If $f(0) = -1$; $f(1) = 1$; and $f(2) = 4$, then the Newton's interpolating equation $f(x) =$ ______

   (a) $\frac{x^3-3x-2}{2}$ (b) $\frac{x^3-3x-2}{2}$

   (c) $x^3 - 3x - 2$ (d) $\frac{x^3-3x-2}{4}$

3. $\left(\frac{dy}{dx}\right)_{x=x_0} =$ _____

(a) $\frac{1}{h}[\frac{\Delta y_0}{2} - \frac{1}{6}\left(\frac{\Delta^3 y_{-1}+\Delta^3 y_{-2}}{2}\right) + \frac{1}{30}\left(\frac{\Delta^5 y_{-2}+\Delta^5 y_{-3}}{2}\right) + \cdots$

(b) $\frac{1}{h^2}[\frac{\Delta y_0+\Delta y_{-1}}{2} - \frac{1}{6}\left(\frac{\Delta^3 y_{-1}+\Delta^3 y_{-2}}{2}\right) + \frac{1}{30}\left(\frac{\Delta^5 y_{-2}+\Delta^5 y_{-3}}{2}\right) + \cdots$

(c) $\frac{1}{h}[\left(\frac{\Delta y_0+\Delta y_{-1}}{2}\right) - \frac{1}{6}\left(\frac{\Delta^3 y_{-1}+\Delta^3 y_{-2}}{2}\right) + \frac{1}{30}\left(\frac{\Delta^5 y_{-2}+\Delta^5 y_{-3}}{2}\right) + \cdots$

(d) $\frac{1}{h^2}[\left(\frac{\Delta y_0+\Delta y_{-1}}{2}\right) - \frac{1}{21}\left(\frac{\Delta^3 y_{-1}+\Delta^3 y_{-2}}{2}\right) + \frac{1}{30}\left(\frac{\Delta^5 y_{-2}+\Delta^5 y_{-3}}{2}\right) + \cdots$

4. $\left(\frac{d^2y}{dx^2}\right)_{x=x_n} =$ _____

(a) $\frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$

(b) $\frac{1}{h}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$

(c) $\frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n - \frac{11}{12}\nabla^4 y_n + \cdots\right]$

(d) $\frac{1}{h^2}\left[\nabla^2 y_n - \nabla^3 y_n - \frac{11}{12}\nabla^4 y_n + \cdots\right]$

5. Trapezoidal rule for double integrals is _____

(a) $I = \frac{hk}{4}[f_{i,j} + 2f_{i+1,j} + f_{i+2,j} + 2f_{i,j+1} + 4f_{i+1,j+1} + 2f_{i+2,j+1} + f_{i,j+2} + 2f_{i+1,j+2} + f_{i+2}]$

(b) $I = \frac{hk}{2}[f_{i,j} + 2f_{i+1,j} + f_{i+2,j} + 2f_{i,j+1} + 4f_{i+1,j+1} + 2f_{i+2,j+1} + f_{i,j+2} + 2f_{i+1,j+2} + f_{i+2,j+2}]$

(c) $I = \frac{hk}{4}[f_{i,j} + 2f_{i+1,j} + f_{i+2,j+2} + 2f_{i,j+1} + 4f_{i+1,j+1} + 2f_{i+2,j+1} + f_{i,j+2} + 2f_{i+1,j+2} + f_{i+2,j+2}]$

(d) $I = \frac{hk}{4}[f_{i,j} + 2f_{i,j+1} + f_{i+2,j+2} + 2f_{i+1,j+1} + 4f_{i+1,j+1} + 2f_{i+2,j+1} + f_{i,j+2} + 2f_{i+1,j+2} + f_{i+2,j+2}]$

6. In Simpson's One-third rule, the principal part of the error in (x_0, x_3) is _____

(a) $-\frac{3}{8}h^5 y''''$ (b) $-\frac{3}{80}h^4 y''''$

(c) $-\frac{3}{80}h^4 y''''$ (d) $-\frac{3}{80}h^5 y''''$

7. $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$ then $y(0.2) =$ _____

(a) 2.2431 (b) 2.1104

(c) 2.4012 (d) 2.5012

8. Euler's iteration formula is _____

(a) $y_{n+1} = y_n + hf(x_n, y_n)$

(b) $y_{n+1} = y_n + f(x_n, y_n)$

(c) $y_n = y_{n+1} + hf(x_n, y_n)$

(d) $y_n = y_{n+1} + f(x_n, y_n)$

9. For solving differential equation in Milne's predictor formula the required number of initial values are _____

(a) 4 (b) 3

(c) 2 (d) 1

10. Milen's predictor formula is _____

(a) $y_{n+1,P} = y_{n-3} + \frac{4h}{3}[y'_{n-2} + y'_{n-1} + 2y'_n]$

(b) $y_{n+1,P} = y_{n-3} + \frac{4h}{3}[y'_{n-2} - y'_{n-1} + 2y'_n]$

(c) $y_{n+1,P} = y_{n-2} + \frac{4h}{3}[y'_{n-2} + y'_{n-1} + 2y'_n]$

(d) $y_{n+1,P} = y_{n-2} + \frac{4h}{3}[y'_{n-2} - y'_{n-1} + 2y'_n]$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Apply Gauss forward interpolation formula to find $y(25)$ for the following data.

x	20	24	28	32
y	2854	3162	3544	3992

Or

(b) Use Lagrange's formula to find the value of $ya + x = 6$ from the following data.

x	3	7	9	10
y	168	120	72	63

12. (a) Given $u_0 = 5; u_1 = 15; u_2 = 57$ and $\frac{du}{dx} = 4$ at $x = 0$ and 72 at $x = 2$. Find $\Delta^3 u_0$ and $\Delta^4 u_0$.

Or

(b) From the following data obtain the first and second derivatives of $\log_{e^x}$ at $x = 500$

x	500	510	520	530	540	550
$y = \log_{e^x}$	6.2146	6.2344	6.2538	6.2729	6.2916	6.3099

13. (a) Find the value $\log 2^{1/3}$ of from $\int_0^1 \frac{x^2}{1+x^3} dx$ using Simpson's $\frac{1}{3}$ rule with $h = 0.25$

Or

(b) A curve passes through the points as given in the table. Find the volume of the solid generated by revolving this area about the $x-$axis.

x	1	2	3	4	5	6	7	8	9
y	0.2	0.7	1	1.3	1.5	1.7	1.9	2.1	2.3

14. (a) Using Taylor's method Solve $\frac{dy}{dx} = 1 + xy$ with $y_0 = 2$ Find $y(0.2)$

Or

(b) Solve $\frac{dy}{dx} = 1 - y$, $y(0) = 0$ using Euler's method. Find y at $x = 0.1$ and $x = 0.2$ Compare the result with results of the exact solution.

15. (a) Using Milne's predictor corrector method find $y(0.4)$ for the differential equation $\frac{dy}{dx} = 1 + xy$, $y(0) = 2$.

Or

(b) Using Adam's Bashforth method find $y(0.4)$ for the differential equation $y' = 1 + xy$, $y(0) = 2$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Given the values

x	5	7	11	13	17
y	150	392	1452	2366	5202

Evalua y_9 using (i) Lagrange's formula (ii) Newton's divided differences formula.

Or

(b) Using Hermite's interpolation find $\sin 1.05$ for the following data

x	1.0	1.1
$y = \sin x$	0.84147	0.89121
$y' = \cos x$	0.5403	0.45360

17. (a) Derive Newton's backward difference formula.

Or

(b) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 51$ from the following data.

x	50	60	70	80	90
y	19.96	36.65	58.81	77.21	94.61

18. (a) Evaluate $\int_0^1 \frac{dx}{1+x}$ using (i) Trapezoidal rule, (ii) Simpson's rule (one third), (iii) Simpson's three eight rule, (iv) Weddley's rule, (v) Find the error in each method by comparing with the actual integration upto 4 places of decimals. Take $h = \frac{1}{6}$ for all cases.

Or

(b) Evaluate $\int_0^1 \int_0^1 xy\, dx\, dy$ using (i) Trapezoidal rule, (ii) Simpson's rule with $h = k = \frac{1}{2}$.

19. (a) Using Euler's method solve $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$. Find $y(0.1)$, $y(0.2)$ and $y(0.3)$. Also find the values by modified Euler's method.

Or

(b) Using Fourth order Runge-kutta method, evaluate the value of y when $x = 1.1$ given that $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$; $y(1) = 1$.

20. (a) Given $\frac{dy}{dx} = \frac{1}{2}(1 + x^2)y^2$ and $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$ Evaluate $y(0.4)$ by Milne's predictor corrector method.

Or

(b) Using Adam's-Bashforth method find $y(4.4)$ given $5xy' + y^2 = 2$, $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ & $y(4.3) = 1.0143$.
